STUDY OF SOME SHORT DISTANCE PARAMETERS IN CURRENTS ASSOCIATED WITH PIONS

by

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DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY KANPUR

MAY, 1975

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A Thesis Submitted
in Partial Fulfilment of the Requirements
for the Degree of
DOCTOR OF PHILOSOPHY

by
VIJAY MOHAN RAVAL

to the

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INDIAN INSTITUTE OF TECHNOLOGY KANPUR

MAY, 1975

C E R T I F I C A T E

Certified that the work presented in this thesis entitled, "Study of Some Short Distance Parameters in Currents Associated with Pions", by V. M. Raval has been carried out under my supervision and that this has not been submitted elsewhere for a degree.

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Thesis entitled, 'Study of Some Short Distance Parameters in Currents Associated with Pions," submitted by VIJAY MOHAN RAVAL in partial fulfilment of the requirements of the Ph.D. Degree to the Department of Physics, Indian Institute of Technology, Kanpur.

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The present work contains some studies of (i) the pion production in electron positron collision processes and (ii) the light cone properties of some hadronic reactions involving pions.

It is known that the infinities present in the field theory give rise to additional terms in the Ward identities which do not appear in their formal derivations. These anomalous terms can not be avoided for abnormal parity amplitudes and their details depend on the choice of the model. Consequently there exist various possibilities for them requiring experimental probes for a decisive choice. The axial vector-vector-vector vertex function in general, possesses anomalies in both the vector and the axial vector Ward identities. It is usually speculated that this vertex has only axial vector anomaly. Using the hard pion technique of Schnitzer and Weinberg we have studied, in second chapter, the effect of these anomalous terms on the neutral pion production in the process $ee \rightarrow ee\pi^0$ for spacelike masses

of photons. Two cases are discussed (i) when vector anomaly is presented and (ii) when it is absent. The numerical results are depicted graphically. The experimental results, when they become available, can possibly identify the presence of vector anomaly.

The second chapter deals with the production of one and two hard pions in the electron positron annihilation processes. Bjorken-Johnson-low theorem and partial conservation of axial vector current have been used to relate these processes at high energy to the short distance structure of the product of two currents. Some useful high energy limits of the inclusive differential cross-sections are obtained. The production of neutral pion and two equal charge pions is found to depend sensitively on the validity of BJL theorem. A possible method for independent measurement of the isovector part of the ratio R is suggested.

It is shown, in the fourth chapter, that the backward region of high energy elastic pion proton scattering is dominated by the singularity structure of the product of two pion source currents near light cone. The available experimental information is used to obtain information about the nature of the dominant operators in the light cone expansion. The plot of $s^3 \frac{d\sigma}{dt}$ against centre of mass scattering angle is found to follow approximately the predicted scaling.

The penult chapter deals with an attempt to connect the scaling properties of pion inclusive spectrum in inelastic pion nucleon collisions to the light cone singularities of the product of pion source currents. Assumptions which are needed to obtain the simple forms of distribution in the fragmentation and pionization regions, have been identified. Within light cone framework the flat rapidity spectrum in the central region could be considered as a consequence of the rapidity independence of the singularity function shown explicitly provided the matrix elements possess weak rapidity dependence.

The final chapter is devoted to a few concluding remarks.

CHAPTER I

INTRODUCTION

Since the uncovering of vector and axial vector nature of weak interactions in the late fifties, the currents have come to play a prominent role in the description of the interactions of hadrons, the strongly interacting particles. Both the electromagnetic as well as the weak interactions seem to be describable in terms of current operators at least at not too high energies. It was observed that the hadronic weak current can be broken up according to the isospin and hypercharge quantum numbers conserved by the strong interactions. This together with the observation that strong interactions are approximately invariant under SU(3) group of transformations has led to a unified basis for describing both the interactions. It was proposed by Cabbibo[1] that all the weak vector currents together with the electromagnetic current should belong to a common SU(3) octet of operators. Similarly, the various weak axial vector currents form an another octet of axial vector operators. This identification is very significant in that the charges associated with the various physical currents can now be considered as the generators of the underlying symmetry group. These charges, which would be conserved when

the symmetry is exact satisfy the Lie algebra of the group. However, in nature, SU(3) symmetry is not exact one and is broken as evidenced by the mass spectrum of the hadrons. Such symmetry breaking effects induce time dependence in the charges and Cabbibo's hypothesis as such loses its significance. This necessitates the introduction of some dynamics into the group structure. Gell-Mann, therefore, proposed[2] that algebraic structure of the underlying symmetry is more basic than the symmetry itself. The time dependence acquired by the charges due to symmetry breaking effects is, therefore, such that the commutation relations between the charges defining the group algebra continue to hold at equal times. These equal time commutation relations can formally be obtained in field theoretic models like σ-model, the free quark model[3] and are given by,

$$[Q_a(x_o),\ V^\mu_b(x)] = i\ f_{abc}\ V^\mu_c\ (x)$$

$$[Q_a(x_o),\ A^\mu_b(x)] = i\ f_{abc}\ A^\mu_c\ (x)$$

$$[Q^5_a(x_o),\ V^\mu_b(x)] = i\ f_{abc}\ A^\mu_c\ (x)$$

$$[Q^5_a(x_o),\ A^\mu_b(x)] = i\ f_{abc}\ V^\mu_c\ (x)$$

where a,b and c = 1,... 8 and the f_{abc} are the structure constants of the algebra of the group SU(3). The generators Q_a and Q^5_a are the vector charges and the axial vector

charges respectively corresponding to the vector current octet V_a^μ and the axial vector current octet A_a^μ. The non-linearity of the algebra fixes the relative scales of the various currents whereas currents measuring charge and hypercharge fix up the overall scale.

One may go a step further and generalise these relations to those between time-space and space-space components of currents. This is not justified because the operator product at the same space time point are not well defined objects. Careful derivation of them yields gradient terms in the delta function known as Schwinger terms[4]. The nature of these non-covariant terms depends on the model and is known very little.

The physical content of the above equal time algebra has been deciphered in essentially two different ways. It has been used to obtain numerous significant results in the form of sum rules and low energy theorems in conjunction with the hypothesis of partial conservation of axial vector current[3]. The derivation of these results uses soft pion approximation which involves extrapolation to the unphysical point of zero pion four momentum. On the other hand, Schnitzer and Weinberg[5] have developed a systematic hard pion technique for investigating the n-point functions of currents. The main assumption in this scheme is the

dominance of the currents by spin zero and spin one mesons. The Ward identities, which form an important ingredient of the technique then provide enough restrictions on the vacuum expectation value of currents to solve the problem. However, in most of these applications naive Ward identities obtained through formal manipulations of divergent expressions are used. This procedure is not justified and has led to incorrent conclusions such as the supression of the neutral pion decay. Adler, first, studied this problem in the framework of perturbation theory[6]. He showed that the infinities inherent in spinar field theory produce, in the Ward identities, well defined extra terms not present in their formal derivations. Since then several authors have studied these identities in a number of spinar models with the same conclusions[7]. In particular, using Pauli and Villar's regularization procedure to define the Green functions of currents, Brown et.al.[7] showed that one can not avoid these additional terms in Ward identities relating abnormal parity amplitudes even after using the freedom of adding to them a polynomial term in momenta. These additional terms have been called anomalous terms. These terms put the renormalisation scheme of gauge theories of weak and electromagnetic interactions in a predicament. They have catalogued the form of the various anomalies in terms of a shift term representing

ambiguity in the assignment of loop momentum and few unknown constants. The shift term depends on the model. Different possibilities exist for the unknown parameters occurring in these anomalies. One particular choice may eliminate an anomalous term from the vector or axial vector Ward identity whereas the other may not. It is, therefore, desirable to have some experimental information regarding them.

The electron positron colliding beam experimental facilities available at places like, SLAC, Frascati etc. offer a unique opportunity to study the much discussed pseudoscalar-vector-vector vertex function which can have anomalous terms in both the vector as well as the axial vector Ward identity. Using the hard pion technique of Schnitzer and Weinberg we have studied the effect of these terms on the π^0 production in the process $ee \to ee\pi^0$ in the region of space-like masses for photons. The phase space calculation is simplified by working in the William-Weizsacker approximation[8] for one of the virtual photons. The behavior of the differential cross-section is found to depend on the various possibilities for the anomalous terms and can possibly show presence of anomaly in the vector Ward identity.

The current algebra scheme furnishes information about the low energy behavior of the Green's function of currents.

The high energy behavior of these functions too is physically relevant due to its bearing on the unitarity and renormalisability of a given theory. Moreover, with high energy accelerators already in operation their asymptotic behavior has acquired additional significance. However, at present, general discussion of high energy behavior of a given theory is not possible. It can be studied only in the framework of renormalised perturbation theory which itself has doubtful validity at such energies. Of late renormalisation group technique has also been used for this purpose. Some years ago, Bjorken and Johnson and Low[9] (BJL) pointed out that the high energy behavior of the time ordered product of two operators can be obtained from the knowledge of the equal time commutator of the two operators and its time derivatives:

$$\lim_{\substack{Q_o \to \infty \\ \bar{Q} \text{ fixed}}} \langle \beta \mid \int e^{iQ.x}\, T(A(x)\; B(o))\, dx \mid \alpha \rangle$$

$$= \frac{i}{Q_o} \int dx\, e^{iQ.x}\, \delta(x_o) \langle \beta \mid [A(x), B(o)] \mid \alpha \rangle$$

$$+ \frac{1}{Q_o^{\,2}} \int dx\, e^{iQ.x}\, \delta(x_o) \langle \beta \mid [\partial_o\, A(x), B(o)] \mid \alpha \rangle$$

$$+ O\left(\frac{1}{Q_o^{\,3}}\right) .$$

However, the divergences present in the theory usually does

not allow the use of cannonical equal time commutation relations. This has become known as the breakdown of the BJL expansion in the perturbation theory[10]. Nevertheless. this expansion can be regarded as the definition of the high energy limit of the time ordered product of two operators. We adopt this point of view in Chapter III but do not use canonical equal time commutators. Its consequences for the single and double hard pion production in the lowest order e^+e^- annihilation processes have been obtained. The differential cross-sections in the central region of the inclusive processes in which only a neutral pion and two equal charge pions are observed have been found to depend sensitively on the validity of the BJL definition. These processes, therefore, can be used to test the same. A method for independent measurement of the isovector part of the ratio $R = \sigma(ee \rightarrow \text{hadrons})/\sigma(ee \rightarrow \mu\mu)$ also emerges.

The equal time algebra of currents makes statements about the singularities of the product of currents on the tip of the light cone. Recently, theoretical interest in the structure of such products near the light cone region was greatly stimulated by the experimental confirmation of the scaling of the deep inelastic electron proton scattering process. This process in the scaling limit is controlled by

the singularity structure of electromagnetic current commutator near light cone region of the coordinate space. This has led to the extension of the short distance expansion of the operator products suggested by Wilson to the light like distances[11]. The tensor and the internal symmetry group structure of the current commutators near the light cone has also been extracted from both the free and the interacting quark models[12]. The basic difference from the equal time algebra is the appearance of unknown bilocal operators. This light cone algebra provides an elegant configuration space description of processes involving a large 'mass' current in the Bjorken scaling limit. It has also corrected few sum rules derived from current algebra.

It is natural to ask if some other high energy limits are also controlled by the singularity structure of the operator products near light cone. It is shown, in Chapter IV, that the backward region of high energy elastic pion nucleon scattering also receives dominant contribution from this region of configuration space. The constraints of keeping the particles on the mass shell compels one to confine to the backward directions only. As in the case of deep inelastic scattering, the available data suggests dominance of the light cone structure of the product of two pion sources by the twist two operators. The scanty and

not too accurate data approximately follows the pattern required by light cone dominance.

Essentially the same technique is used to study the pion inclusive process in the limiting fragmentation and the pionization regions in the penult chapter. Its behavior in these regions is related to the light cone singularities of the product of pion source currents. Assumptions, which are needed to obtain the simple forms of distribution function, have been identified. We sum up in the final chapter with a few concluding remarks.

CHAPTER II

ANOMALOUS WARD IDENTITIES AND π^0 PRODUCTION

Colliding electron (positron) beams offer a very potential source of information about electromagnetic and strong interactions. Laboratories with such a facility have furnished first clean study of the well known vector mesons. Of late, a few new resonances have also been predicted. This has created much incentive for theoretical investigations of the one photon annihilation process, $e^+ + e^- \to \gamma \to$ hadrons shown in Fig. 1(a). However, it was realised sometime ago that this annihilation process together with other higher order ones decreases with increase in beam energy but the fourth order process shown in Fig. 1(b) is logarithmically enhanced due to the possibility of almost real photon exchange[13]. This two photon annihilation mechanism of particle production is, therefore, expected to dominate at high energy. This opens up a whole area of new possibilities and information to be gained from such processes. In particular, it can be utilized for investigating the photon-photon-hadron vertex in the kinematical region where virtual photons possess space like masses. With this in view we consider here the production of neutral pion in the process $ee \to ee\pi^0$ which explores the much discussed pseudoscalar-vector-vector vertex. The present discussion

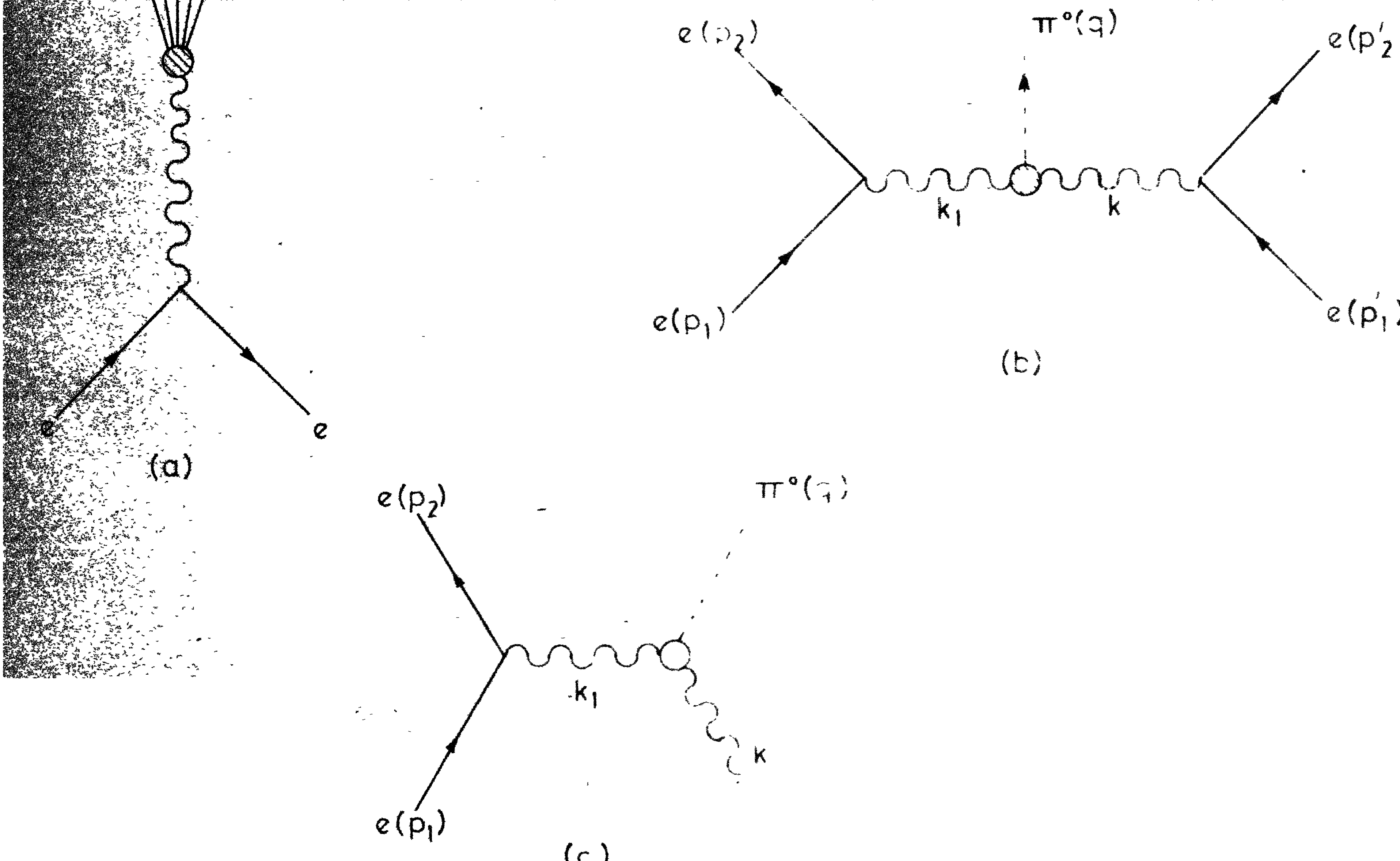


Fig. 1

is confined to the kinematical region where dominance of the amplitude by vector mesons makes sense.

The relevant hadron vertex was shown by Veltman and Sutherland[14] to vanish in the conventional current algebraic approach, which assumes the proportionality of axial vector current divergence to the pion field. This contradicts the experimentally observed two photon decay of neutral pion. Subsequently, Bell and Jackiw[15] pointed out that the decay is not suppressed in σ-model which has PCAC built in it. The resolution of this contradiction is found in the breakdown of the usual arguments used in writing Ward identities. Namely, the Schwinger terms in the equal time commutators and the divergence of the sea-gull term covariantizing the time ordered products are consistently ignored on the presumption of their cancelling each other. Such a cancellation was conjectured by Feynman. It has been shown that in general this need not happen[16]. This immediately raises doubt about the validity of Ward identities obtained through formal manipulations - known as naive Ward identities (NWI). Adler, first studied this problem within the framework of perturbation theory in the context of spinor axial vector current[6]. It was found that the axial current does not satisfy the NWI and also its divergence is not the usual expression obtained by formal manipulations of field equations.

The reason for this was traced to the presence of linearly divergent closed loop triangle diagram which prohibits shift in the integration variable needed for naive derivation to be correct. This stimulated the study of Ward identities in various models with the inference that shift terms, known as anomalous terms, do appear in NWI. In particular, Brown et.al.[7] concluded that these terms can not be avoided even after using the freedom of adding a polynomial term to the regularised Green's function. They have catalogued the various abnormal parity amplitudes together with the explicit forms of their anomalies upto some constants. Constants depend on the choice of the model, and their experimental study should be capable of discerning among the various possibilities.

The AVV vertex has been shown to have in general, anomalies in both the axial vector and vector Ward identities. There are speculations that this vertex has only axial vector anomaly. But it has not been established that when vector currents acquire mass the contribution of the vector anomaly will be meagre. We study, in this chapter, the effect of both axial vector and vector anomaly on π^0 production in the two photon process $ee \rightarrow ee\pi^0$ [17].

We begin, in the next section, with the anomalous axial vector and vector Ward identities for the three point

function $< 0 \mid T A^a_\mu (x) V^b_\nu (y) V^c_\lambda (0) \mid 0 >$. Using hard meson technique with the assumption that the entire hadronic electromagnetic current is composed of the linear combination of the known neutral vector meson fields, we obtain the PVV vertex in terms of eight unknown parameters. We take quark model value for one of them. In Section 3, we see that the neutral vector meson decays and the π^o-decay enable us to fix up the remaining parameters. Here an expression for the slope of the π^o form factor has also been obtained. In Section 4, we derive an expression for the differential cross-section and present numerical results. The last section is devoted to the discussion of the results.

2.2 $\gamma - \gamma - \pi^o$ VERTEX FUNCTION:

Consider the reaction,

$$e^-(p_1) + e^+(p_1') \to e^-(p_2) + e^+(p_2') + \pi^o(q) \qquad (2.1)$$

which proceeds predominantly according to the Feynman diagram of Fig. 1(b). Lorentz covariance and parity conservation restrict the form of the hadron vertex to,

$$T_{\mu\nu} = i \int dx\, e^{ik_1 . x} < 0 \mid T J_\mu (x) J_\nu(o) \mid \pi^o(q) >$$

$$= F(q^2, k_1^2, k^2)\, \epsilon_{\mu\nu\alpha\beta}\, k^\alpha k_1^\beta \qquad (2.2)$$

where J_μ is the electromagnetic current and k, k_1 momenta of the virtual photons. The functional form of F is obtained using the well known Schnitzer-Weinberg technique[5]. Its main assumption is that the vector and axial vector currents are dominated by spin-0 and spin-1 mesons. This is imposed in the form of the simple momentum dependence of the proper vertices, together with the requirement that the spectral functions of the propagators are dominated by the one meson states. Ward identities then determine the unknown coefficients in the proper vertices.

Let us define,

$$M(q,k)^{abc}_{\mu\nu\lambda} = \int dxdy\ e^{-iq.x-ik.y} \langle o| TA^a_\mu(x) V^b_\nu(y) V^c_\lambda(o)|o\rangle \quad (2.3)$$

$$M(q,k)^{abc}_{\nu\lambda} = \int dxdy\ e^{-iq.x-ik.y} \langle o| T\partial^\mu A^a_\mu(x) V^b_\nu(y) V^c_\lambda(o)|o\rangle \quad (2.4)$$

Now using equal time algebra,

$$\delta(x_o-y_o)\ [A^a_o(x),\ V^b_\mu(y)] = if^{abc} A^c_\mu(x)\ \delta(x-y).$$

$$\delta(x_o-y_o)\ [V^a_o(x),\ A^b_\mu(y)] = if^{abc} A^c_\mu(x)\ \delta(x-y)$$

$$\delta(x_o-y_o)\ [V^a_o(x),\ V^b_\mu(y)] = if^{abc} V^c_\mu(x)\ \delta(x-y)$$

and charge conjugation invariance Ward identities satisfied by $M(q,k)_{\mu\nu\lambda}$ can be written, suppressing SU(3) indices, as,

$$q^\mu M(q,k)_{\mu\nu\lambda} = -i\, M(q,k)_{\nu\lambda} + \Delta(AVV)_{\nu\lambda} \quad (2.5)$$

$$k^\nu M(q,k)_{\mu\nu\lambda} = \Delta_V(AVV)_{\mu\lambda} \quad (2.6)$$

$$k_1^\lambda M(q,k)_{\mu\nu\lambda} = \Delta_V(AVV)_{\mu\nu} \quad (2.7)$$

Here Δ and Δ_V are the axial vector and vector anomalous terms[7], respectively, given by,

$$\Delta(A^a V^b V^c)_{\nu\lambda} = (2b_1 - 3y)\, d^{abc}\, \epsilon_{\nu\lambda\alpha\beta}\, k^\alpha k_1^\beta \qquad (2.8)$$

$$\Delta_V(A^a V^b V^c)_{\mu\lambda} = b_1\, d^{abc}\, \epsilon_{\mu\lambda\alpha\beta}\, k^\alpha\, k_1^\beta \qquad (2.9)$$

These terms do not occur in the formal derivation of the Ward identities. Here y represents the surface term arising when a shift of integration variable is carried out in the linearly divergent integral representation of the three point function. It is zero if there are no divergences in the theory. The quantity b_1 is the coefficient of the polynomial term which can always be added to the regularised vertex function. Both these quantities are model dependent.

Since the neutral pion decays into an isoscalar and an isovector photon indices b and c take the values 3, 8 and o. The operators V_μ^8 and V_μ^o are, respectively, proportional to the hypercharge current and the baryon current. Assuming vector meson dominance[18] we can then write,

$$V_\lambda^8 = f_Y' (\cos\theta_Y\, m_\phi^2\, \phi_\lambda - \sin\theta_Y\, m_\omega^2\, \omega_\lambda) \qquad (2.10)$$

$$V_\lambda^o = f_B' (\cos\theta_B\, m_\omega^2\, \omega_\lambda + \sin\theta_B\, m_\phi^2\, \phi_\lambda) \qquad (2.11)$$

where $f_Y' = \sqrt{3}/2 f_Y$ and $f_B' = \sqrt{3/2}/f_B$. f_Y and f_B are the hypercharge and the baryon coupling constants. θ_Y is the mixing angle between ω and ϕ fields corresponding to the

hypercharge current. θ_B is the similar angle for the baryon current.

Using equations (2.10) and (2.3) we define the proper vertices $\Gamma^{\mu\nu\lambda}_{\phi,\omega}$ and $\Gamma^{\nu\lambda}_{\phi,\omega}$ as follows:

$$M(q,k)^{ab8}_{\mu\nu\lambda} = d^{ab8}\ f'_Y[g_A^{-1}\ g_V^{-1}\ \Delta^A(q)_{\mu\mu'}\ \Delta^V(k)_{\nu\nu'}\ \{\cos\theta_Y\ m_\phi^2\ \Delta^\phi(k_1)_{\lambda\lambda'}$$
$$\Gamma_\phi(q,k)^{\mu'\nu'\lambda'} - \sin\theta_Y \Delta^\omega(k_1)_{\lambda\lambda'}\ \Gamma_\omega(q,k)^{\mu'\nu'\lambda'}\}] +$$
$$+\ d^{ab8}\ m_\pi^{-2}\ f'_Y\ q^\mu\ [\Delta_\pi(q)\ g_V^{-1}\ \Delta^V(k)_{\nu\nu'}\ \{\cos\theta_Y\ m_\phi^2$$
$$\Delta^\phi(k_1)_{\lambda\lambda'}\ \Gamma_\phi(q,k)^{\nu'\lambda'} - \sin\theta_Y\ m_\omega^2\ \Delta^\omega(k_1)_{\lambda\lambda'}\ \Gamma_\omega(q,k)^{\nu'\lambda'}\}]$$

(2.12)

$$M(q,k)^{ab8}_{\mu\nu} = d^{ab8}\ f'_Y\ [\Delta_\pi(q)\ g_V^{-1}\ \Delta^V(k)_{\nu\nu'}\ \{\cos\theta_Y\ m_\phi^2\ \Delta^\phi(k_1)_{\lambda\lambda'}$$
$$\Gamma_\phi(q,k)^{\nu'\lambda'} - \sin\theta_Y\ m_\omega^2\ \Delta^\omega(k_1)_{\lambda\lambda'}\ \Gamma_\omega(q,k)^{\nu'\lambda'}\}] \qquad (2.13)$$

Here $k_1 = -q-k$ and $\Delta^V(k)_{\mu\nu}$ and $\Delta^A(k)_{\mu\nu}$ are the covariant spin one parts of the vector and axial vector propagators. $\Delta^{\omega,\phi}(k_1)_{\mu\nu}$ is the vector meson propagator. These are defined in Appendix A. After some straight forward algebra we get the Ward identities in terms of the Γ-vertices.

$$\cos\theta_Y\ m_\phi^2\ \Delta^\phi(k_1)_{\lambda\lambda'}\ \Gamma_\phi(q,k)^{\nu'\lambda'} - \sin\theta_Y\ m_\omega^2\ \Delta^\omega(k_1)_{\lambda\lambda'}\ \Gamma_\omega(q,k)^{\nu'\lambda'}$$
$$= i\ g_A^{-1}\ \Delta_\pi(q)^{-1}\ q^\mu\ \Delta^A(q)_{\mu\mu'}\ \{\cos\theta_Y\ m_\phi^2\ \Delta^\phi(k_1)_{\lambda\lambda'}\ \Gamma_\phi(q,k)^{\mu'\nu'\lambda'} -$$
$$\sin\theta_Y\ m_\omega^2\ \Delta^\omega(k_1)_{\lambda\lambda'}\ \Gamma_\omega(q,k)^{\mu'\nu'\lambda'}\} - i\ f'^{-1}_Y\ g_V\ \Delta_\pi(q)^{-1}\ \Delta^V(k)^{-1}_{\nu\nu'}\ \ \Delta(AVV)_{\nu\lambda}$$

(2.14)

$$k^{\nu}\, \Delta^{V}(k)_{\nu\nu'}\, \{\cos\theta_Y\, m_{\phi}^2\, \Delta^{\phi}(k_1)_{\lambda\lambda'}\, \Gamma_{\phi}(q,k)^{\mu'\nu'\lambda'} - \sin\theta_Y\, m_{\omega}^2\, \Delta^{\omega}(k_1)_{\lambda\lambda'}$$

$$\Gamma_{\omega}(q,k)^{\mu'\nu'\lambda'}\} = f_Y'^{-1}\, g_A\, g_V\, \Delta^{A}(q)^{-1}_{\mu\mu'}\, \Delta(AVV)_{\mu\lambda} \tag{2.15}$$

$$k_1^{\lambda}\ \{\cos\theta_Y\, m_{\phi}^2\, \Delta^{\phi}(k_1)_{\lambda\lambda'}\, \Gamma_{\phi}(q,k)^{\mu'\nu'\lambda'} - \sin\theta_Y\, m_{\omega}^2\, \Delta^{\omega}(k_1)_{\lambda\lambda'}$$

$$\Gamma_{\omega}(q,k)^{\mu'\nu'\lambda'}\} = f_Y'^{-1}\, g_A\, g_V\, \Delta^{A}(q)^{-1}_{\mu\mu'}\, \Delta^{V}(k)^{-1}_{\nu\nu'}\, \Delta_V(AVV)_{\mu\nu} \tag{2.16}$$

Notice that (2.14) contains four unknown Γ-functions. To solve for $\Gamma^{\mu\nu}_{\omega,\phi}$ in terms of $\Gamma^{\mu\nu\lambda}_{\omega,\phi}$ another equation, analogous to Eq. (2.14), is obtained using equations (2.3) and (2.11). This can directly be written from equation (2.14) by making the following replacements,

$$\begin{aligned} f_Y' &\rightarrow f_B' \\ \cos\theta_Y &\rightarrow \sin\theta_B \\ \sin\theta_Y &\rightarrow -\cos\theta_B \end{aligned} \tag{2.17}$$

$$\sin\theta_B\, m_{\phi}^2\, \Delta^{\phi}(k_1)_{\lambda\lambda'}\, \Gamma_{\phi}(q,k)^{\lambda\lambda'} + \cos\theta_B\, m_{\omega}^2\, \Delta^{\omega}(k_1)_{\lambda\lambda'}\, \Gamma_{\omega}(q,k)^{\lambda\lambda'}$$

$$= i\, g_A^{-1}\, \Delta_{\pi}(q)^{-1}\, q^{\mu}\, \Delta^{A}(q)_{\mu\mu'}\ \{\sin\theta_B\, m_{\phi}^2\, \Delta^{\phi}(k_1)_{\lambda\lambda'}\, \Gamma_{\phi}(q,k)^{\mu'\nu'\lambda'} +$$

$$\cos\theta_B\, m_{\omega}^2\, \Delta^{\omega}(k_1)_{\lambda\lambda'}\, \Gamma_{\omega}(q,k)^{\mu'\nu'\lambda'}\} - i\, f_B'^{-1}\, g_V\, \Delta_{\pi}(q)^{-1}\, \Delta^{V}(k)^{-1}_{\nu\nu'}\Delta(AVV)_{\nu\lambda} \tag{2.18}$$

Using equations (2.14) and (2.18) finally we get,

$$-i\, \Gamma_{\omega,\phi}(q,k)^{\nu'\lambda'} = g_A^{-1}\, \Delta_{\pi}(q)^{-1}\, q^{\mu}\, \Delta^{A}(q)_{\mu\mu'}\, \Gamma_{\omega,\phi}(q,k)^{\mu'\nu'\lambda'}$$

$$- g_V\, \Delta_{\pi}(q)^{-1}\, \Delta^{V}(k)^{-1}_{\nu\nu'}\, \Delta^{\omega,\phi}(k_1)^{-1}_{\lambda\lambda'}\, m^{-2}_{\omega,\phi} \times$$

$$\sec(\theta_B - \theta_Y)\, \lambda_{\omega,\phi}\, \Delta(AVV)_{\nu\lambda} \tag{2.19}$$

where,

$$\lambda_\omega = f_B'^{-1} \cos\theta_Y - f_Y'^{-1} \sin\theta_B$$

$$\lambda_\phi = f_Y'^{-1} \cos\theta_B + f_B'^{-1} \sin\theta_Y$$

The structure of the vertices appearing on the r.h.s. of (2.19) is dictated by the vector Ward identities (2.15) and (2.16). Writing similar equations for c = o we solve for them obtaining

$$k_{\nu'}\Gamma_{\omega,\phi}(q,k)^{\mu'\nu'\lambda'} = -\lambda_{\omega,\phi}\, g_A\, g_V^{-1} \sec(\theta_B - \theta_Y)\, m_V^2\, m_{\omega,\phi}^{-2}\, \Delta^A(q)^{-1}_{\mu\mu'} \times \Delta^{\omega,\phi}(k_1)^{-1}_{\lambda\lambda'}\, \Delta_V(AVV)_{\mu\lambda} \tag{2.20}$$

$$k_{1\lambda'}\, \Gamma_{\omega,\phi}(q,k)^{\mu'\nu'\lambda'} = \lambda_{\omega,\phi}\, g_A\, g_V\, \Delta^A(q)^{-1}_{\mu\mu'}\, \Delta^V(k)^{-1}_{\nu\nu'} \sec(\theta_B - \theta_Y)\, \Delta_V(AVV)_{\mu\nu} \tag{2.21}$$

Lorentz covariance restricts $\Gamma_{\mu\nu\lambda}$ to have the form

$$\Gamma_{\mu\nu\lambda}(k,k_1) = \gamma_1\, \varepsilon_{\mu\nu\lambda\sigma}\, k^\sigma + \gamma_2\, \varepsilon_{\mu\nu\lambda\sigma}\, k_1^\sigma + \gamma_3\, k_\mu\, \varepsilon_{\nu\lambda\sigma\tau}\, k^\sigma\, k_1^\tau + \gamma_4\, k_{1\mu}\, \varepsilon_{\nu\lambda\sigma\tau}\, k^\sigma\, k_1^\tau + \gamma_5\, k_\nu\, \varepsilon_{\mu\lambda\sigma\tau}\, k^\sigma\, k^\tau + \gamma_6\, k_{1\nu}\, \varepsilon_{\mu\lambda\sigma\tau} k^\sigma\, k_1^\tau \tag{2.22}$$

where $\gamma_j = \gamma_j\, (q^2, k^2, k_1^2)$

We now assume meson dominance of the propagator spectral functions and impose the restriction that the

momentum dependence of the proper vertices be as smooth as possible. This requires as we shall see later, γ_j, j = 3, ..., 6 to be independent of momenta. Using equations (2.19) - (2.22) we obtain,

$$\gamma_1^{\omega,\phi} = -b_1 \lambda_{\omega,\phi} \sec(\theta_B-\theta_Y) g_{A_1}^{-1} g_\rho^{-1} (q^2-m_{A_1}^2)(k^2-m_\rho^2) \tag{2.23}$$

$$-\gamma_2^{\omega,\phi} + k^2 \gamma_5^{\omega,\phi} + \frac{1}{2}(q^2 - k^2 - k_1^2)\gamma_6^{\omega,\phi}$$
$$= b_1 \lambda_{\omega,\phi} \sec(\theta_B-\theta_Y) m_{\omega,\phi}^{-2} m_\rho^2 g_{A_1}^{-1} g_\rho^{-1} (q^2-m_{A_1}^2)(k_1^2-m_{\omega,\phi}^2) \tag{2.24}$$

$$- i\,\Gamma_{\omega,\phi}(q,k)^{\nu'\lambda'} = \{-g_{A_1} m_{A_1}^{-2} \Delta_\pi(q)^{-1} (-\gamma_1^{\omega,\phi} + \gamma_2^{\omega,\phi} + \frac{1}{2}(q^2+k^2-k_1^2)\gamma_3^{\omega,\phi}$$
$$+ \frac{1}{2}(q^2+k_1^2-k^2)\gamma_4^{\omega,\phi}) + (2b_1 - 3y)\lambda_{\omega,\phi} \sec(\theta_B-\theta_Y) m_{\omega,\phi}^{-2} g_\rho^{-1} \Delta_\pi(q)^{-1}$$
$$(k_1^2 - m_{\omega,\phi}^2)(k^2-m_\rho^2)\}\varepsilon^{\nu'\lambda'\alpha\beta} k_\alpha k_{1\beta} \tag{2.25}$$

Substituting in (2.25) for γ_1 and γ_2 from (2.23)-(2.24) we get,

$$-i\,\Gamma_{\omega,\phi}(q,k)^{\nu'\lambda'} = \{-g_{A_1} m_{A_1}^{-2} \Delta_\pi(q)^{-1}(q^2 \Gamma_1^{\omega,\phi} + k^2 \Gamma_2^{\omega,\phi} + k_1^2 \Gamma_3^{\omega,\phi})$$
$$-b_1 \lambda_{\omega,\phi} \sec(\theta_B-\theta_Y) g_\rho^{-1} m_{A_1}^{-2} \Delta_\pi(q)^{-1} (q^2-m_{A_1}^2)(k^2 m_{\omega,\phi}^2 - k_1^2 m_\rho^2)/m_{\omega,\phi}^2$$
$$+ (2b_1-3y)\lambda_{\omega,\phi} \sec(\theta_B-\theta_Y) g_\rho^{-1} m_{\omega,\phi}^{-2} \Delta_\pi(q)^{-1}(k^2-m_\rho^2)(k_1^2-m_{\omega,\phi}^2)\}\varepsilon^{\nu'\lambda'\alpha\beta} k_\alpha k_{1\beta}$$
$$\tag{2.26}$$

where Γ's are linear combinations of γ_j, $j = 3,\ldots 6$ and smoothness assumption requires them to be independent of momenta. It then follows that these little gammas should be constants. We now have the complete structure of the function $M_{\nu\lambda}$, Eq. (2.13), which is proportional to the pion decay amplitude $T_{\mu\nu}$. This gives us

$$F(q^2,k^2,k_1^2) = \frac{(2b_1-3y)}{3f_\pi} e^2 + \frac{b_1 e^2 f'_Y (q^2-m_{A_1}^2)}{3f_\pi m_{A_1}^2 (k^2-m_\rho^2)} \sec(\theta_B-\theta_Y)$$

$$\times \left\{\frac{(k_1^2 m_\rho^2 - k^2 m_\phi^2)}{(k_1^2-m_\phi^2)} \lambda_\phi \cos\theta_Y - \frac{(k_1^2 m_\rho^2 - k^2 m_\omega^2)}{(k_1^2 - m_\omega^2)} \lambda_\omega \sin\theta_Y\right\}$$

$$+ \frac{e^2 g_\rho f'_Y}{3f_\pi (k^2-m_\rho^2)} \left\{-g_{A_1} m_{A_1}^{-2} \left[(q^2 \Gamma_1^\phi + k^2 \Gamma_2^\phi + k_1^2 \Gamma_3^\phi) \frac{\cos\theta_Y m_\phi^2}{(k_1^2-m_\phi^2)}\right.\right.$$

$$\left.\left. - (q^2 \Gamma_1^\omega + k^2 \Gamma_2^\omega + k_1^2 \Gamma_3^\omega) \frac{\sin\theta_Y m_\omega^2}{(k_1^2-m_\omega^2)}\right]\right\} + (k \leftrightarrow k_1) \qquad (2.27)$$

This gives

$$F(0,0,0) = \frac{(2b_1-3y)e^2}{3f_\pi}$$

i.e. in the soft pion limit the two photon decay of the neutral pion is allowed. In the absence of axial vector anomaly this decay will be suppressed and we obtain the result of Sutherland namely, $F(0,0,0) = 0$. It should be

noted in Eq. (2.27) that the structure dependent part depends only on the vector anomaly b_1. Therefore, from the π-decay alone one cannot conclude the absence of vector anomaly. Its effects can be observed in processes involving at least one of the two photons off the mass-shell.

For $q^2 = k^2 \approx 0$ and $m_\omega^2 \approx m_\rho^2$ $F(q^2,k^2,k_1^2)$ becomes,

$$F(0,0,k_1^2) \approx \frac{(2b_1-3y)}{3f_\pi} e^2 + \frac{b_1 e^2 \, f'_Y \sec(\theta_B-\theta_Y)}{3f_\pi} k_1^2 \, \lambda_\phi \cos\theta_Y$$

$$\times \left\{\frac{1}{(k_1^2 - m_\phi^2)} - \frac{1}{(k_1^2-m_\rho^2)}\right\} + \frac{e^2 \, f'_Y \, g_\rho \, g_{A_1} \, m_{A_1}^{-2}}{3f_\pi \, m_\rho^2} k_1^2 \left[\cos\theta_Y \left\{\right.\right.$$

$$\left.\left.\frac{m_\phi^2 \, \Gamma_3^\phi}{(k_1^2-m_\phi^2)} + \frac{m_\rho^2 \, \Gamma_2^\phi}{(k_1^2-m_\rho^2)}\right\} - \frac{\sin\theta_Y \, m_\rho^2}{(k_1^2-m_\rho^2)} (\Gamma_3^\omega + \Gamma_2^\omega)\right] \qquad (2.28)$$

The constants b_1 and y are model dependent. Their relative strength depends on the choice of the model. In Han - Nambu model, the value of y alone accounts for the π^0 - decay, thereby indicating the absence of vector anomaly i.e. $b_1 = 0$. Its quark model value predicts the decay width nine times smaller than the observed value. Hence it

requires substantial contribution to the AVV vertex from the vector anomaly. To estimate maximal effect of vector anomaly we take the quark model value for y:

$$y = \frac{1}{24\pi^2}$$

Also we assume smooth extrapolation of the function F from the unphysical point $q^2 = 0$ to its mass shell value which requires $q^2 \Gamma_1^{\omega,\phi}$ to have small variations in the region of extrapolation. We approximate it by its value at the lower end of the region. This eliminates $\Gamma_1^{\omega,\phi}$ from Eq. (2.27).

2.3 NEUTRAL VECTOR MESON DECAYS AND THE CONSTANTS Γ :

The three point function $M_{\mu\nu}^{abc}$, which we have obtained above, also describes the decays $\phi \to \rho\pi$, $\phi \to \pi\gamma$, $\omega \to \pi\gamma$, $\omega \to \rho\pi$ and $\rho \to \pi\gamma$. We use these decays to determine the constants $\Gamma_2^{\omega,\phi}$ and $\Gamma_3^{\omega,\phi}$. Using (2.13) and (2.26) and writing electromagnetic current J_μ as,

$$J_\mu = e\,[V_\mu^3 + \frac{1}{\sqrt{3}} V_\mu^8]$$

we obtain for the decay constants,

$$f_{\omega\rho\pi} = -\frac{g_{A_1}}{\sqrt{3}\,f_\pi}\,\frac{(m_\rho^2\,\Gamma_2^\omega + m_\omega^2\,\Gamma_3^\omega)}{m_{A_1}^2} \tag{2.29}$$

$$f_{\omega\pi\gamma} = -\frac{e}{\sqrt{3}\,f_\pi}\,[b_1\,\lambda_\omega\,\sec\,(\theta_B-\theta_Y) + \frac{g_\rho g_{A_1}\,m_\omega^2}{m_{A_1}^2\,m_\rho^2}\,\Gamma_3^\omega] \tag{2.30}$$

$$f_{\rho\pi\gamma} = \frac{e}{3f_\pi}\,[\frac{b_1\,m_\rho^2}{g_\rho} + \frac{g_{A_1}\,m_\rho^2}{m_{A_1}^2}\,f_Y'\,(\Gamma_2^\omega\,\sin\,\theta_Y - \Gamma_2^\phi\,\cos\,\theta_Y)] \tag{2.31}$$

$$f_{\phi\rho\pi} = \frac{g_{A_1}}{\sqrt{3}\,f_\pi}\,\frac{(m_\rho^2\,\Gamma_2^\phi + m_\phi^2\,\Gamma_3^\phi)}{m_{A_1}^2} \tag{2.32}$$

$$f_{\phi\pi\gamma} = \frac{e}{\sqrt{3}\,f_\pi}\,[b_1\,\lambda_\phi\,\sec\,(\theta_B-\theta_Y) + \frac{g_\rho g_{A_1}\,m_\phi^2}{m_\rho^2\,m_{A_1}^2}\,\Gamma_3^\phi] \tag{2.33}$$

The derivation of above relations has been sketch in Appendix B. We observe that only radiative decays receive contribution from the vector anomaly. Now experimentally observed widths of the decays $\phi \to \pi\gamma$ and $\phi \to \rho\pi$ are very small. This we take into account by assuming that $\phi \to \rho - \pi$ vertex vanishes. Later we shall see that our results are not sensitive to this assumption. Equations (2.32) and (2.33) then determine Γ_2^ϕ and Γ_3^ϕ in terms of b_1 and other parameters:

$$\Gamma_3^\phi = -\frac{b_1\,m_\rho^2\,m_{A_1}^2\,\lambda_\phi\,\sec\,(\theta_B-\theta_Y)}{g_\rho\,g_{A_1}\,m_\phi^2} \tag{2.34}$$

$$\frac{\Gamma_2^{\phi}}{\Gamma_3^{\phi}} = - \frac{m_{\phi}^2}{m_{\rho}^2} \tag{2.35}$$

The decays $\omega \to \rho\pi$ and $\omega \to \pi\gamma$ fix up the constants $\Gamma^{\omega}_{2,3}$. For mixing parameters we take Oakes and Sakurai type current mixing[19]. The coupling constants g_ρ and g_{A_1} are obtained from the current algebra results[20]:

$$g_\rho^2 = 2m_\rho^2 \; f_\pi^2$$

and

$$g_{A_1}^2 = g_\rho^2$$

Using above information we obtain eight sets of values for Γ's. Four of them are eliminated by using the constraint that the $\rho \to \pi\gamma$ decay lies within the present experimental limit i.e.

$$\Gamma(\rho \to \pi\gamma) < 0.56 \text{ MeV}$$

We list the remaining four sets in table 1 together with the values of Γ's for the case $b_1 = 0$. We now have all the information about the structure function $F(q^2, k^2, k_1^2)$ so we can proceed to compare with its experimental measurement. For the experimental comparison we consider the $\pi^o\gamma\gamma$ vertex function occurring in a) $\pi^o \to \gamma + (e^+ + e^-)$ and b) $e^+ e^- \to e^+ e^- \pi^o$.

The neutral pion apart from the dominant two photon mode also decays via internal conversion process $\pi^0 \to \gamma + e^+ e^-$ initiated by the $\pi^0 \gamma \gamma$ vertex. Therefore, in the kinematical region $k^2 = 0$ and $4m_e^2 < k_1^2 < m_\pi^2$ the function F should describe the Dalitz distribution of the lepton pair. The form factor effects in this region are described by a slope parameter a_π defined as

$$F(m_\pi^2, xm_\pi^2, o) = F(o, o, o)\ (1 + a_\pi\ x)$$

where $x = k_1^2/m_\pi^2$. The value of a_π is characteristic of the nature of $\pi^0 - \gamma$ interaction. From (2.27) we obtain the following expression for a_π:

$$a_\pi = \frac{m_\pi^2\ f'_y}{(2b_1-3y)} \left[\frac{g_\rho^2}{m_{A_1}^2\ m_\rho^2} \{\sin\theta_Y(\Gamma_2^\omega+\Gamma_3^\omega) - \cos\theta_Y\ (\Gamma_2^\phi + \Gamma_3^\phi)\} - \right.$$

$$\left. b_1 \sec(\theta_B - \theta_Y)\ \{\lambda_\phi \cos\theta_Y(\frac{1}{m_\phi^2} - \frac{1}{m_\rho^2}) - \lambda_\omega \sin\theta_Y\ (\frac{1}{m_\omega^2} - \frac{1}{m_\rho^2})\}\right]$$

This gives us the following two possible values of a_π:

$$|a_\pi| = 0.012 \qquad \text{and} \qquad |a_\pi| = 0.014$$

The sign of a_π depends on the relative signs of Γ's which can not be ascertained. The vector meson formalism and the dispersion theoretic approach with resonance

dominance predict the values 0.032 and 0.046 respectively for the slope parameter[21]. The available experimental values are:

$$a_\pi = -0.24 \pm 0.12^{22} \qquad \text{and} \qquad a_\pi = -0.15 \pm 0.10^{23}$$

$$a_\pi = 0.01 \pm 0.11^{24}$$

The discrepancy between the theory and experiment may be due either to the contribution from higher mass states or non-resonant contributions. But in view of the large errors involved there is a need to obtain more accurate values of a_π before some definite conclusions can be drawn about $\pi^0-\gamma$ dynamics.

2.4 DIFFERENTIAL CROSS SECTION FOR THE PROCESS $ee \to ee\pi^0$:

In this section, we consider π^0 - production in the process (2.1), which explores the function $F(q^2,k^2,k_1^2)$ for spacelike values of photon masses. The reaction proceeds via the Feynman diagram shown in Fig. 1(b). We simplify the phase space calculation by assuming that one of the electron beams acts as a source of the virtual photon beam and only the transverse polarisation provide the dominant contribution. For then the equivalent photon approximation[8] can be invoked to obtain the following relationship:

$$\frac{d^2\sigma}{dE'\ d\cos\theta'} = \int \frac{dk_o}{k_o} N(k_o) \frac{d^2\ \sigma_{e\gamma\to e\pi^\circ}}{dE'\ d\cos\theta'} \qquad (2.36)$$

where,

E = beam energy; k_o = photon energy; $E_2 = E - k_o$

E' = energy of the electron scattered at the angle θ'

and,

$$N(k_o) = \frac{\alpha}{\pi} \left[\frac{E^2 + E_2^2}{E^2} (\ln E/m_e - 1/2) + \frac{(E - E_2)^2}{2E^2} \times \right.$$

$$\left. \left(\ln \frac{2E_2}{E-E_2} + 1\right) + \frac{(E - E_2)^2}{2E^2} \ln \frac{2E}{(E - E_2)} \right]$$

The derivation of Eq. (2.36) has been sketched in the Appendix C. Now the matrix element for the process $e\gamma \to e\pi^o$ can be written as,

$$M = e\ \bar{u}(p_2)\ \gamma^\mu\ u(p_1) \frac{1}{k_1^{\ 2}}\ T_\mu \qquad (2.37)$$

where $k_1 = p_1 - p_2$ and T_μ is given by (2.2). Using (2.36) and (2.27) we obtain the following expression for the differential cross section in the centre of mass frame,

$$\frac{d^2\sigma_{ee\to ee\pi^\circ}}{dE'\ d\cos\theta'} = \frac{\pi^2\ \alpha^3\ E'}{\lambda E|k_1^2|} |F(0,0,k_1^2)|^2\ N(k_o)\ [4E^2+E'^2(1+\cos\theta')^2]$$

where,

$$\lambda = (2E - E' - E' \cos \theta'); \quad k_o = -k_1^2/2\lambda$$

and $F(o, o, k_1^2)$ is given by Eq. (2.28).

In Fig. 3, we have plotted this differential cross section as a function of E' for the two values 1.4 GeV and 1.6 GeV of the beam energy E and for the scattering angle $\theta' = 20^o$. This choice explores the hadron vertex for the photon mass in the range $0.13 < -k_1^2$ (GeV^2) < 0.27. To get an idea of the momentum dependence we have also plotted in Fig. (2), $|F(o, o, k_1^2)|^2$ and the corresponding $|F|^2$ obtained from the π^o-decay width.

2.5 DISCUSSION:

From the table 1 we note that $\Gamma^\omega_{2,3}$ are order of magnitude larger than Γ^ϕ's for the ϕ - mesons and therefore dominate the last term in the Eq. (2.28). Therefore, as mentioned earlier, our function F is not very sensitive to the values of $\Gamma^\phi_{2,3}$. So we do not expect significant modification of our results when the fact that the decay $\phi \to \rho\pi$ does take place is taken into account. Furthermore, it is the sum $\Delta\Gamma^\omega$ of Γ^ω_2 and Γ^ω_3 which is important and not their absolute values. From the table it is seen that there are only two different values of $\Delta\Gamma^\omega$ for the four sets, which either add to the contribution of the axial vector anomaly or subtract from it. Thus, as seen from Fig. (3) we

Table 1

Values of the quantities $\Gamma' = \Gamma\, m_\pi^2$ and the sum $\Delta\Gamma^\omega = |\Gamma_2'^\omega + \Gamma_3'^\omega|$ for the two cases $b_1 \neq 0$ and $b_1 = 0$.

No.	b_1	$\Gamma_2'^\phi$	$\Gamma_3'^\phi$		$\Gamma_2'^\omega$	$\Gamma_3'^\omega$	$\Delta\Gamma^\omega$
1	0.050	0.156	-0.088	(a)	-2.78	1.64	1.14
				(b)	-0.146	1.22	1.06
2	-0.037	-0.12	0.066	(a)	-1.40	0.34	1.06
				(b)	2.58	-1.44	1.14
3	0.0	0.0	0.0		± 2.00	∓ 0.88	1.12

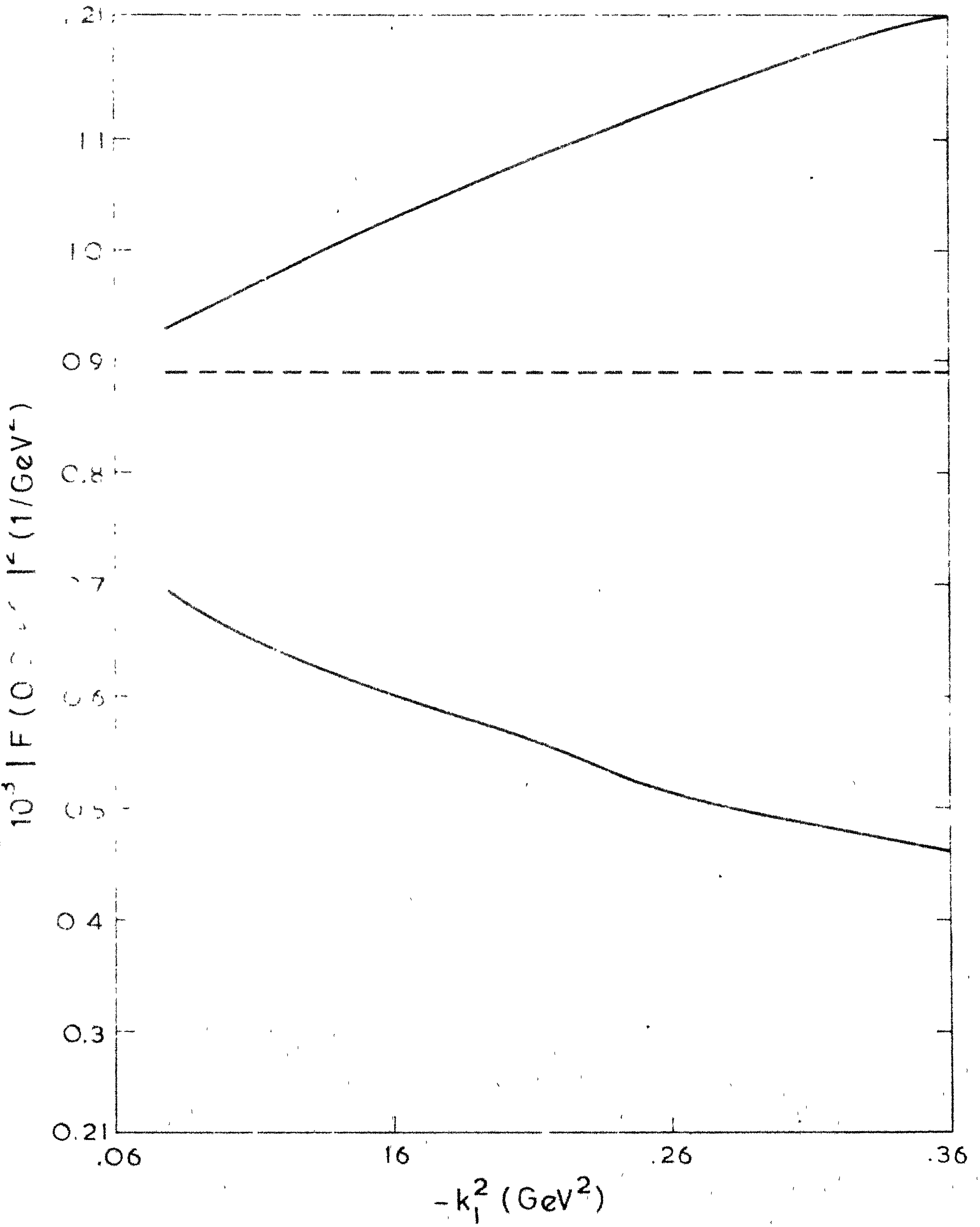
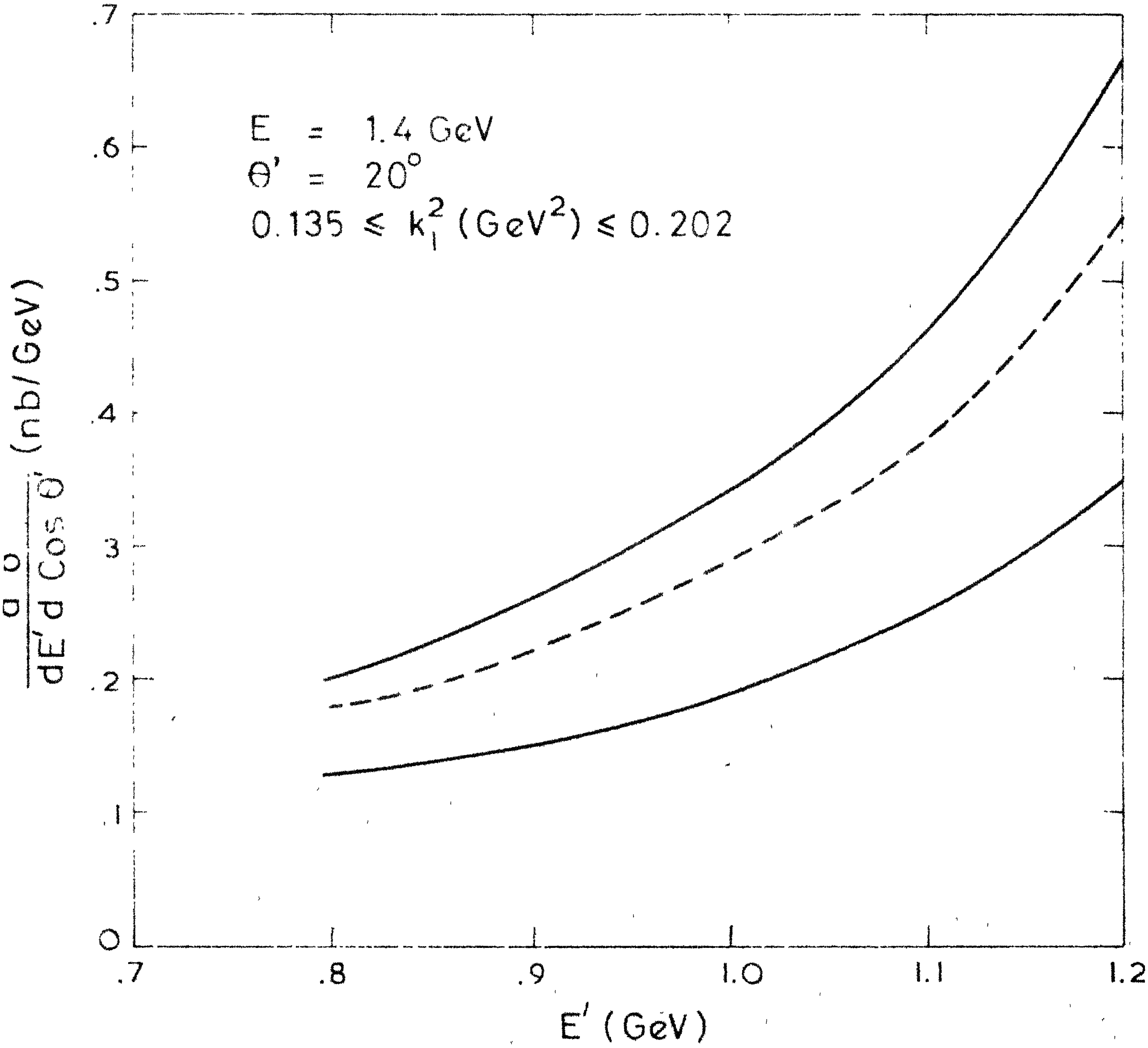


Fig. 2



E = 1.4 GeV
θ' = 20°
0.135 ≤ k_1^2 (GeV²) ≤ 0.202
$\frac{d\sigma}{dE' d \cos\theta'}$ (nb/GeV)
E' (GeV)
.7
.6
.5
.4
.3
.2
.1
0
.7
.8
.9
1.0
1.1
1.2

Fig. 3a

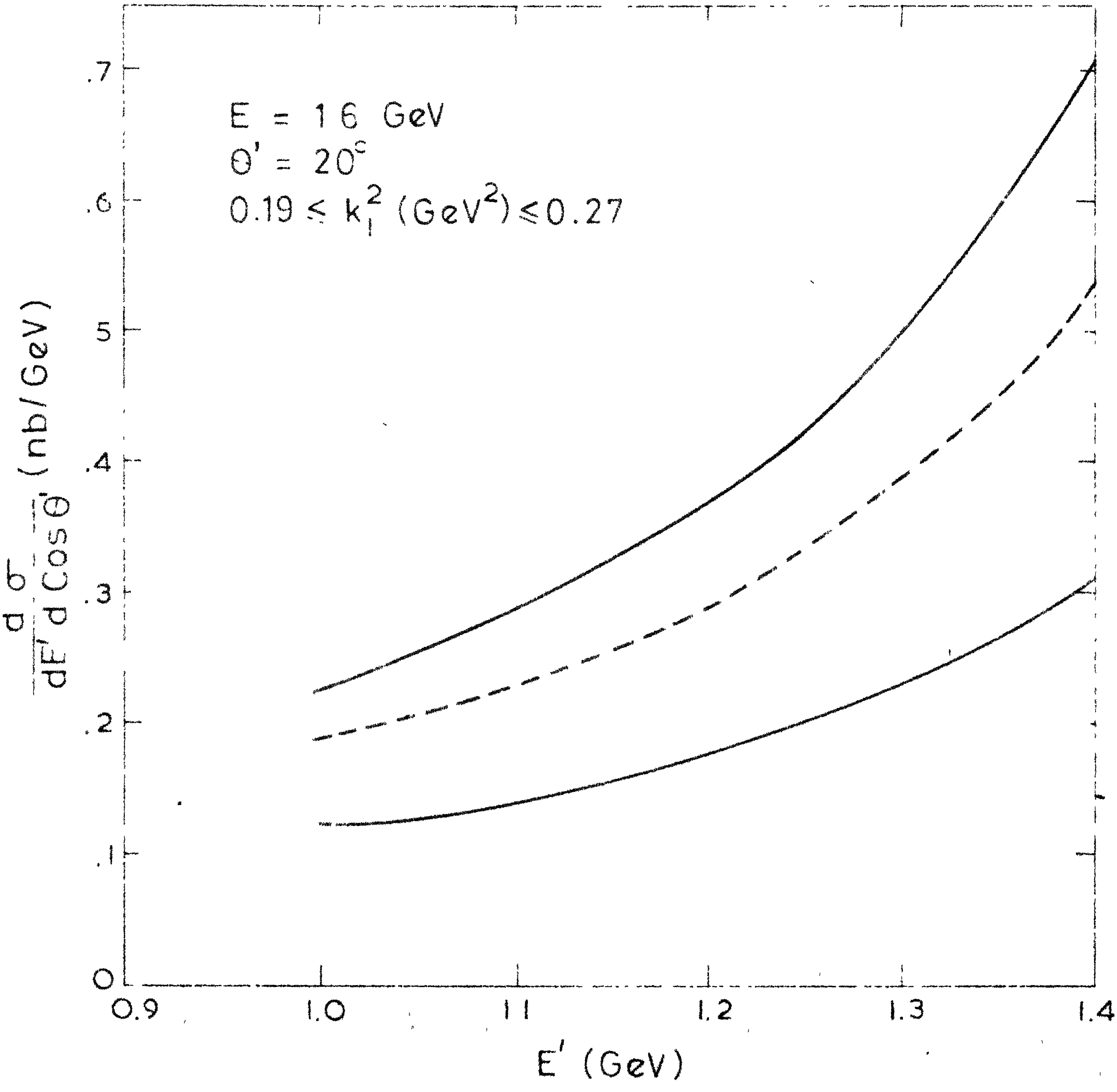

E = 16 GeV
θ' = 20°
0.19 ≤ k_1^2 (GeV²) ≤ 0.27
.7
.6
.5
.4
.3
.2
.1
0
$\frac{d\sigma}{dE' \, d\cos\theta'}$ (nb/GeV)
0.9
1.0
1.1
1.2
1.3
1.4
E' (GeV)

Fig. 3b

obtain two different predictions, at the same photon mass, for the differential cross section which lie on either side of its value with no structure dependence.

We see from Figs. (2) and (3) that for small values of k_1^2 the structure effects, as expected, do not show up very much. But as the photon acquires a significant mass, the dependence of the hadron vertex on it becomes important. With the increase in the photon mass the deviation from the curve without structure dependence increases. The reason is the presence in the function F, Eq. (2.28), of the factor $k_1^2/(k_1^2 - m_x^2)$ which increases with k_1^2.

The upward trend, relative to no structure dependence part, in the differential cross-section should be contrasted with the situation in vector meson dominance model where the differential cross-section will always decrease with the increase of k_1^2. Moreover, in VMD, the decrease with k_1^2 will not be as rapid as in our case because of the absence of the above mentioned factor.

For the case $b_1 = 0$, which coincides with the Han-Nambu model, the values of the constants Γ's change substantially, see table 1. In this model, the sum $\Delta\Gamma^{\omega}$ for different sets of values of $\Gamma^{\omega}_{2,3}$ always adds to the contribution of the axial vector anomaly y. This, in turn, increases the differential cross section for all the sets,

in contrast with the quark model predictions. The value of $\Delta\Gamma^{\omega}$ in this model is found to be nearly equal to its value in the quark model. Therefore, the upper curves in Fig. (3) almost coincide in the two models making distinction between the two difficult.

Equation (2.36) is valid strictly in the case of the unobserved electron scattered only in the near forward directions. Therefore, the differential cross section computed using (2.36) would differ from the actual value. On the basis of numerical considerations[25] it has been found that the double equivalent photon approximation generally under estimates the exact result by 10 to 15 percent. Since in writing (2.36) only one photon has been treated in this approximation we expect our results to reasonably approximate the actual ones.

The detection of one of the outgoing electrons in coincidence with the produced hadron eliminates the background due to the annihilation processes. The only other process which can interfer with the two photon mechanism is the one where both photons exchanged are time like. The contribution of this diagram is expected to be quite small and therefore has been neglected. Such a process was, in fact, analysed by Young[26] using a similar technique but with an altogether different motivation. The main emphasis

in his work is to look for the observable effects of axial vector anomaly. Since this anomaly in the Han Nambu model accounts for almost entire neutral pion decay, he assumed the absence of any anomalous term in the vector Ward identity. Here we have tried to see the effects of the vector anomaly which is allowed from the general considerations. In this regard the choice of the quark model is significant since here vector anomaly gives substantial contribution to the PVV vertex enabling one to study its maximal effect. As the quark model value of axial vector anomaly gives π^0 decay width nine times smaller than the experimental value, Young conjectures against smooth extrapolation in q^2 in this model. Obviously this is true only in the absence of vector anomaly. The radiative decays of the neutral vector mesons which fix up the constants Γ's have been found to depend on the vector anomaly b_1 in contrast to Young's work. He obtains total cross section of the order of 10^{-37} cm^2 supporting the expectation that contribution to π^0 production from the time like region should be small. The contribution from the space like photons, considered here, is much larger and is further enhanced at high energies. The differential cross section obtained is of the order of 10^{-33} cm^2 which should be easier to measure. Furthermore the differential cross section

in the quark model exhibits, as mentioned above, a feature distinct from that in Han Nambu model namely, the downward trend relative to no structure dependent part.

Now experimentally if the differential cross section lies above the curve with no structure dependence, one can not distinguish between the two possibilities for b_1 in the proposed reaction. However, if it shows a downward trend and goes faster than expected on the basis of VMD it should be an indication of the presence of the vector anomaly.

CHAPTER III

HARD PION PRODUCTION IN ELECTRON POSITRON ANNIHILATION

The importance of the study of soft pion production in electron positron annihilation reaction was first emphasised by Pais and Treiman[27]. These authors showed that processes,

$$e^+ + e^- \rightarrow \text{soft pion} + \text{anything} \tag{3.1a}$$

and

$$e^+ + e^- \rightarrow \text{two soft pions} + \text{anything} \tag{3.1b}$$

in the lowest order in electromagnetism, can furnish valuable information on otherwise experimentally inaccessible spectral functions associated with the strangeness conserving weak axial vector current and the isovector electromagnetic current. These spectral functions bear important information about the structure of the interactions. Their analysis based on the notions of current algebra and partial conservation of axial vector current, involves delicate energy dependent extrapolations of the amplitude to the unphysical region of zero four momentum pion. This technique is, therefore, applicable only to annihilation processes devoid of final state baryon antibaryon pairs thus avoiding the contribution of soft pion emissions. The mass of the

virtual photon is, therefore, restricted to $k^2 < 4$ (baryon mass)^2.

In this chapter, we study the production of hard pions in the above processes in the high energy region - where the virtual photon mass is not restricted instead is allowed to go to infinity. It is of interest to discuss these, since, as we shall see, they provide information on the high energy dynamical parameters appearing in the short distance operator product expansions of $A_\mu(x)\, A_\nu(o)$ and $V_\mu(x)\, V_\nu(o)$. In particular the isovector part of the ratio R defined by,

$$R = \sigma(ee \to \text{hadrons})/\sigma(ee \to \mu\mu)$$

which is not separable in the total annihilation process, can possibly be measured in the inclusive process in Eq. (3.1b). The ratio R, in general, should depend on the energy square k^2 but is expected to approach a constant as $k^2 \to \infty$. Present experiments indicate that the ratio R is still on the rise. However, one striking feature which has emerged is that it has already exceeded the constant values predicted by various quark models[28].

Our analysis of reactions, Eq. (3.1) involves partial conservation of axial vector current together with the BJL expansion of the time ordered product of two currents

Consequences are obtained which can be used to test the soundness of BJL expansion[29].

3.2 ONE PION PRODUCTION:

Let us, first, consider the following reaction,

$$e^+(p_1) + e^-(p_2) \to \pi^a(q) + X(P)$$

in the centre of mass system i.e. $\vec{p}_1 + \vec{p}_2 = 0$. X denotes the unobserved hadron system. The inclusive cross section in the lowest order in electromagnetic coupling is given by,

$$\omega_q \frac{d\sigma^a}{d^3q} = \frac{32\pi^3\alpha^2}{k^6} l^{\mu\nu} \quad W^a_{\mu\nu} \quad (q,k) \qquad (3.2)$$

where ω_q energy of pion; $\alpha = 1/137$ fine structure constant

the lepton vertex tensor $l^{\mu\nu} = (p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - \frac{1}{2}k^2 g^{\mu\nu}$

virtual photon momentum $k = p_1 + p_2$

and

$$W^a_{\mu\nu} \quad (q,k) = \frac{1}{2\pi} \int dx \, e^{-ik.x} \sum_X \langle o|J_\nu(o)|\pi^a(q)X\rangle$$
$$\langle X \, \pi^a(q)|J_\mu(x)|o\rangle \qquad (3.3)$$

with J_μ as hadronic electromagnetic current.

Consider the integral,

$$I^a_\mu = \int dx \, e^{-ik.x} \langle X\pi^a(q)|J_\mu(x)|o\rangle \qquad (3.4)$$

in Eq. (3.3). Using the Lehman-Symanzik-Zimmerman reduction formula to reduce in the final state pion and then PCAC,

$$\partial^{\mu} A^{a}_{\mu} = f_{\pi} \mu^{2} \phi^{a}$$

it becomes,

$$I^{a}_{\mu} = \frac{i}{\sqrt{2(2\pi)^{3}}\, f_{\pi}} \int dx\, dy\, e^{-ik.x+iq.y}$$

$$[iq^{\nu} \langle X| TA^{a}_{\nu}(y) J_{\mu}(x)|o\rangle - \delta(x_{o}-y_{o}) \langle X|[A^{a}_{o}(y), J_{\mu}(x)]|o$$

(3.5)

Now change of integration variable and the use of the equal time commutator,

$$\delta(x_{o})\, [A^{a}_{o}(x/2), J_{\mu}(-x/2)] = - A^{a}_{\mu}(o)\, \delta(x) + \ldots$$

give Eq. (3.5) the following form,

$$I^{a}_{\mu} = \frac{i}{\sqrt{2(2\pi)^{3}}\, f_{\pi}} [\langle X| A^{a}_{\mu}(o)|o\rangle + iq^{\nu} \int dx\, e^{iQ.x}$$

$$\langle X| TA^{a}_{\nu}(x/2)\, J_{\mu}(-x/2)|o\rangle]\, (2\pi)^{4}\, \delta(P + q - k)$$

(3.6)

Here,

$$Q = \frac{1}{2}(q + k), \quad P = k - q,$$

a is the isotopic index for pion and A^{a}_{μ} $\Delta S = o$ weak axial vector current. For convenience the produced pion is considered

to be π^- so that a = 1 + i2. In arriving at Eq. (3.6) extrapolation to the point $q^2 = \mu^2 = o$ has been done. Such an extrapolation is not serious one for the present region of interest where $k^2 >> \mu^2$, the pion mass. It should be noted that $q^2 = o$ does not necessarily imply $q = o$.

Now we have,

$$Q_o = \frac{1}{2}(q_o + k_o), \quad \vec{Q} = \frac{1}{2}\vec{q} = -\frac{1}{2}\vec{P}$$

Therefore, BJL limit, which is $Q_o \to \infty$ with $\vec{Q}$ and all hadron momenta fixed, is physically accessible if the beam energy $E \to \infty$ and pion momentum $\vec{q}$ is held fixed. So the BJL expansion can be used for the time ordered product term in Eq.(3.6) which then becomes,

$$I_\mu^a \underset{\substack{Q_o \to \infty \\ \overline{Q}\text{ fixed}}}{\approx} \frac{i}{\sqrt{2(2\pi)^3}\, f_\pi} [\langle X|A_\mu^a(o)|o\rangle + iq^\nu\{\frac{1}{Q_o}\int dx\, e^{iQ\cdot x}$$
$$\delta(x_o) \langle X \mid [A_\nu^a(x/2), J_\mu(-x/2)]|o\rangle\}$$
$$+ o(1/Q_o^2)]\ (2\pi)^4\, \delta(P + q - k)$$

The BJL theorem involves an infinite sum and in order that it makes sense what one should make sure is that matrix elements of all the equal time commutators appearing in the expansion are finite. At present such a thing can be checked only in perturbation theory where calculations have revealed the break down of the expansion at the first stage itself.

However, at high energy perturbation theoretic results should be taken with some caution. The finite scaling limit confirmed in the deep inelastic region of lepton hadron scattering process, in fact, makes one optimistic in this regard as it shows that at least the lowest dimensional operators that occur in the equal time electromagnetic current commutators possess finite nucleonic matrix elements[30]. This encourages one to assume the validity of BJL expansion and look for the consequences which can be put to experimental tests. Following Gross and Treiman[30] we suppose that the expansion makes sense at least to the orders Q_o^{-1} and Q_o^{-2}. Thus the first term, if nonvanishing controls the asymptotic behaviour as $Q_o \to \infty$. Using Eq. (3.4) one obtains,

$$\langle X \, \pi^a (q) \mid J_\mu(o) \mid o \rangle \approx \frac{i}{\sqrt{2(2\pi)^3} \; f_\pi} \langle X \mid A^a_\mu (o) | o \rangle \tag{3.7}$$

Equation (3.3) then becomes,

$$W^a_{\mu\nu} (q,k) \approx \frac{1}{2(2\pi)^4 \; f^2_\pi} \int dx \; e^{-i(k-q).x} \langle o | A^{a\dagger}_\nu (o) \; A_\mu (x) | \; o \rangle \tag{3.8}$$

$$= \frac{1}{2(2\pi)^4 \; f^2_\pi} \int dx \; e^{-ik_o x_o - i\bar{q}.\bar{x}} \langle o | [A^{a\dagger}_\nu(o), \; A_\mu(x)] | o \rangle$$

We see immediately that as $k_o \to \infty$ only the short distance singularity structure of the product of axial vector currents is important. Moreover, what enters here is only its c-number part. This can be easily obtained in a free quark model and is given by,

$$A^+_\mu (x)\, A^-_\nu (o) \triangleq R_A\, (2\partial_\mu \partial_\nu - g_{\mu\nu} \Box) \frac{1}{\pi^4\, (x^2 - i\epsilon x_o)^2}$$

$$+ \text{ operator terms} \qquad (3.9)$$

The value of the constant R_A depends on the underlying quark structure of hadrons and is of interest here.

Substituting (3.9) in (3.8) and using the relation,

$$\int dx\, e^{iq.x} \frac{1}{(x^2 - i\epsilon x_o)^2} = -\,\theta(q^2)\, \pi^3$$

we obtain,

$$W^a_{\mu\nu} \quad (q,k) \simeq \frac{R_A}{(2\pi)^5\, f_\pi^2} [2(k-q)_\mu \, (k-q)_\nu - (k-q)^2\, g_{\mu\nu}] \qquad (3.10)$$

As such $W^a_{\mu\nu}$ is not gauge invariant. However the lepton vertex tensor $l^{\mu\nu}$ preserves the electromagnetic gauge invariance by picking up only that part of $W^a_{\mu\nu}$ which vanishes when contracted with either k_μ or k_ν. This is because the tensor $l^{\mu\nu}$ satisfies the following properties

$$l^{\mu\nu} \quad k_\mu \quad k_\nu \quad = \quad 0$$

and

$$l^{\mu\nu} \quad (k_\mu \; q_\nu \; + k_\nu \; q_\mu) \; = \; 0$$

Using $W^a_{\mu\nu}$, given by Eq. (3.10), the inclusive cross section, Eq. (3.2), becomes,

$$\omega_q \frac{d\ \sigma^a}{d^3 q} \simeq \frac{R_A \ \alpha^2}{\pi^2 \ f_\pi^2} \frac{1}{k^4} \left[(k^2 - 2q.k) + q_o^2 \ (1 - \cos^2 \theta) \right]$$

where θ is the angle of emission of the pion relative to the beam axis. Now the angular integration can be easily done and so after some rearrangements finally we get,

$$\lim_{\substack{k^2 \to \infty \\ q_o \text{ fixed}}} \omega\, k^2 \frac{d\ \sigma^\pm}{d(q.k)} \simeq \frac{4\alpha^2}{\pi\ f_\pi^2} R_A \tag{3.11}$$

where $\omega = k^2/q.k$. This equation provides a means of obtaining experimentally the short distance parameter R_A ($= 2R_A^{(3)}$) in the single charged pion inclusive process at high energy. $R_A^{(3)}$ is the corresponding quantity for the product A^3_μ (x) A^3_ν (o). The relation of $R_A^{(3)}$ to an observable quantity established above supplements a result of Crewther and thus makes possible an independent measurement of the Adler's anomalous constant S through the following result arrived

at by him[31].

$$S = 8\,K\,R_A^{(3)} \tag{3.12}$$

Here K is another short distance parameter occurring in the antisymmetric part of the short distance operator product expansion of two electromagnetic currents. It is determinable either from Bjorken's sum rule for polarised deep inelastic electroproduction or from the high energy limit of the cross-section for $e^+ + e^- \to \pi^0\,\mu^+\,\mu^-$ with $\vec{q}$ fixed[32]. The relation (3.12) is quite significant as it relates high energy cross sections to an anomaly of low energy theorem. The value of S obtained from this relation should be compared with the PCAC result S = 0.5.

Since for the emission of a neutral pion the first term in Eq. (3.6) vanishes, the result to order α^2 is

$$\lim_{\substack{k^2 \to \infty \\ q_0 \text{ fixed}}} \omega\, k^2 \frac{d\sigma^0}{d(q.k)} = 0 \tag{3.13}$$

The contribution, if any, now will come from the time ordered product term in Eq. (3.6). The above result can, therefore, be used as a test of the validity of the BJL expansion. The extent of the break down of this relation to order α^2 will determine how good or bad the approximation is. This result also applies for any individual channel

$\pi^o + X$. Therefore, in the central region of the inclusive reaction the number of annihilation events involving neutral pions should decrease with the increase in the beam energy. The triangle graph anomaly in the divergence of the neutral axial vector current will contribute to Eq. (3.13) terms of the order of α^3.

3.3 TWO PION PRODUCTION:

Now let us consider the production of two hard pions,

$$e^+ (p_1) + e^- (p_2) \to \pi^a(q) + \pi^b(q') + X (P)$$

The inclusive cross section for this process can be written as,

$$\omega_{q'} \omega_q \frac{d\, \sigma^{ab}}{d^3q\, d^3q'} = \frac{8\pi^2\alpha^2}{k^6} l^{\mu\nu} \quad W^{ab}_{\mu\nu} \quad (q,q',k) \qquad (3.14)$$

where ω_q and $\omega_{q'}$ are the energies of pions and the tensor,

$$W^{ab}_{\mu\nu} (q,q',k) = \int dx\, e^{-ik.x} \sum_X \langle o| J_\nu(o)|\pi^a\pi^b X.\rangle$$

$$\langle X\, \pi^a\pi^b \mid J_\mu(x) \mid o \rangle \qquad (3.15)$$

Consider again the integral,

$$I_\mu = \int dx\, e^{-ik.x} \langle X\, \pi^a\, \pi^b \mid J_\mu(x) \mid o \rangle \qquad (3.16)$$

As in the single pion production case, contracting the final state pion π^a and using BJL expansion together with PCAC

we obtain the following relation analogous to Eq. (3.7),

$$I_\mu \underset{\substack{Q_o \to \infty \\ \frac{Q_o}{\overline{Q}} \text{ fixed}}}{\approx} \frac{i}{\sqrt{2(2\pi)^3}\, f_\pi} < X\, \pi^b\, (q')|A_\mu^a\, (o)|o >(2\pi)^4\, \delta(P+q+q'-k)$$

$$= \frac{i}{\sqrt{2(2\pi)^3}\, f_\pi} \int dx\, e^{-i(k-q).x} < X\, \pi^b\, (q')|A_\mu^a\, (x)|o > \tag{3.17}$$

Now reducing the second pion π^b also in Eq. (3.17) one obtains,

$$I_\mu \approx -\frac{1}{2(2\pi)^3\, f_\pi^2} \int dx\, dy\, e^{i(q-k).x+iq'.y} \,.$$

$$[iq'^\lambda < X|TA_\lambda^b\, (y)\, A_\mu^a\, (x)|o > -\delta(x_o-y_o)<X|\,[A_o^b(y), A_\mu^a\, (x)]\,|o> \tag{3.18}$$

With the change of integration variables and use of the equal time commutator,

$$\delta(x_o)\; [A_o^b(x/2),\, A_\mu^a\, (-x/2)] \;=\; i\, f_{bac}\, V_\mu^c\, (o)\, \delta(x) + \ldots$$

Eq. (3.18) assumes the following form,

$$I_\mu \approx \frac{1}{2(2\pi)^3\, f_\pi^2} [\; i\, f_{bac} < X\, |\, V_\mu^c\, (o)|o > + iq'^\lambda \int dx\, e^{iQ'.x}$$

$$< X\, |\, TA_\lambda^b\, (x/2)\, A_\mu^a\, (-x/2)\, |\, o >] (2\pi)^4\, \delta(P+q+q'-k) \tag{3.19}$$

where,

$$Q' = \frac{1}{2}(k - q + q'); \quad P = k - q - q'$$

and

$$Q'_o = \frac{1}{2}(k_o - q_o + q'_o); \quad \overline{Q}' = \frac{1}{2}(\vec{q}' - \vec{q})$$

So if the momenta $\overline{q}$ and $\overline{q}'$ of the two pions are kept fixed one can again use the BJL expansion for the time ordered product in Eq. (19). Neglecting its contribution in the limit $Q'_o \to \infty$, we obtain the relation,

$$\langle X \pi^a(q) \pi^b(q') \mid J_\mu(o)|o\rangle$$

$$\simeq \frac{1}{2(2\pi)^3 f_\pi^2} \, i \, f_{bac} \langle X \mid V^c_\mu(o)|o\rangle \qquad (3.20)$$

Using this relation for pions of different charges Eq.(3.15) assumes the following form,

$$W^{+-}_{\mu\nu}(q,q',k) \simeq \frac{1}{(2\pi)^6 f_\pi^4} \int dx \, e^{-i(k-q-q').x}$$

$$\langle o| V^3_\nu(o) \, V^3_\mu(x)|o\rangle \qquad (3.21)$$

The exponential, neglecting the finite energies of the pions, becomes exp $\{-ik_o x_o - (\overline{q} + \overline{q}').\overline{x}\}$ in the centre of mass frame. Therefore, again, as $k_o \to \infty$, what is involved is the isovector part $R_V^{(3)}$ of the ratio R, defined by the following short distance operator product expansion,

$$V_\mu^3(x)\, V_\nu^3(o) \;\hat{=}\; R_V^{(3)}(2\partial_\mu\partial_\nu - g_{\mu\nu}\Box)\,\frac{1}{\pi^4(x^2 - i\epsilon x_o)^2}$$

$$+ \text{ operator terms} \qquad (3.22)$$

Using this in Eq. (3.21) we obtain from (3.14),

$$\omega_q\, \omega_{q'}\, \frac{d^2\sigma^{+-}}{d^3q\, d^3q'} \simeq \frac{2\alpha^2}{(2\pi)^4\, \pi\, f_\pi^4}\, \frac{R_V^{(3)}}{k^4}\, [(k-Q'')^2$$

$$- 2q_o q_o'\,(1 - \cos\theta_1) + \{Q_o'^2 - (q_{11} + q_{11}')^2\}]$$

where

$$Q'' = \frac{1}{2}(q + q')\;;\quad q_{11} = |\bar{q}|\cos\theta;\quad q_{11}' = |\bar{q}'|\cos\theta$$

θ **and** θ' are the angles which pion momenta make with the beam axis. θ_1 represents the angle between the two outgoing pions. In the limit $k^2 \to \infty$ all the angular dependence disappears and after some rearrangements we finally get,

$$\lim_{\substack{k^2 \to \infty \\ q_o,\, q_o' \text{ fixed}}} \omega\omega'\, k^2\, \frac{d\,\sigma^{+-}}{d(q.k)\, d(q'.k)} \simeq \frac{2\alpha^2}{\pi^3 f_\pi^4}\, R_V^{(3)} \qquad (3.23)$$

where $\omega' = k^2/q'.k$. Above equation shows that high energy two pion events in the central region where pions possess finite energies can provide information on otherwise unaccessible isovector part $R_V^{(3)}$ of the ratio R.

For the emission of two equal charge pions since the structure constant in Eq. (3.20) vanishes we obtain the following prediction,

$$\lim_{\substack{k^2 \to \infty \\ q_o, q_o' \text{ fixed}}} \omega\, \omega' \; k^2 \frac{d^2 \sigma^{eq.ch.}}{d(q.k)\; d(q'.k)} \simeq 0 \qquad (3.24)$$

provided the contribution of the terms in BJL expansion neglected in arriving at (3.20) is small. Therefore this relation can also serve the purpose of checking the validity of the BJL theorem i.e. the finiteness of the matrix elements of the equal time commutators appearing in the expansion. This result is in fact, valid for any individual channel and if checked in experiments should point out the usefulness of the concepts used here.

Now the constraints imposed on R_A and R_V by the chiral invariance and the octet dominance of the electromagnetic current can be used to obtain useful relations between the high energy limits of the various differential cross sections. The constants R_A and R_V are related by chiral invariance at short distances which implies $R_A = R_V$. This gives us the following equality,

$$\lim_{\substack{k^2 \to \infty \\ q_o \text{ fixed}}} \omega \; k^2 \frac{d\, \sigma^{+-}}{d(q.k)} = (2\pi f_\pi)^2 \lim_{\substack{k^2 \to \infty \\ q_o, q_o' \text{ fixed}}} \omega\, \omega' \; k^2 \frac{d^2\, \sigma^{+-}}{d(q.k)\; d(q'.k)}$$

The extent to which the above relation is obeyed experimentally should show the goodness of the invariance at short distances.

If the electromagnetic current J_μ is a pure octet operator one should have $3R \simeq 4 R_V^3$. But if it has components belonging to higher representations of SU(3) then $3R > 4R_V^3$. This implies,

$$\lim_{k^2 \to \infty} k^2 \sigma(k^2) \geq \frac{8}{9} (\pi f_\pi)^4 \lim_{\substack{k^2 \to \infty \\ q_o, q_o' \text{ fixed}}} \omega \omega' k^2 \frac{d^2 \sigma^{+-}}{d(q.k)\, d(q'.} \tag{3.26}$$

It should be possible to experimentally verify this inequality.

3.4 DISCUSSION:

All the results obtained above depend on the absence of q-number Schwinger terms from the various equal time commutators and the reliability of the BJL expansion. The operator Schwinger terms plague all the calculations of current algebra. The nature of all these terms, besides model, depends on the kind of interaction and the procedure one adopts for the calculation of the equal time commutator. To settle the question of their nature, therefore, one must turn towards the experiments. Unfortunately unambiguous

experimental numbers on semi-inclusive processes in $e^+ e^-$ annihilation are not yet available and consequently little is known about the matrix elements used in the above expression. This makes the estimation of their effect very difficult. In the case of neutral axial vector current, however, the contribution of such terms is expected to be of the order of α^3. This is because the electromagnetic interactions modify the axial vector vertex by terms of the order of α. We may, therefore, hope that at least to the order α^2 which is of interest to us their contribution will be small and hence may be neglected. The reliability of the BJL expansion can be tested using relations (3.13) and (3.24) which imply that these cross-sections should approach to zero at least as fast as $1/Q_0^2$ in the high energy limit $k^2 \to \infty$. The use of PCAC at high energy is of course questionable. However, since the quantities q.k and q'.k enter only as the ratios $q.k/k^2$ and $q'.k/k^2$ the accuracy of using PCAC here will be of the same order as that at low energies.

Thus, so far as the BJL theorem proves to be a reliable tool and the contribution of Schwinger terms is small equations (3.11) and (3.23) reveal that pion production in the central region in the high energy inclusive $e^+ e^-$ annihilation reaction can provide information on the c-number portion of the short distance expansion of the

products $A_\mu(x)\, A_\nu(o)$ and $V_\mu(x)\, V_\nu(o)$. In particular, direct experimental determination of the isovector part of the much discussed ratio R seems possible. The determination of these c-numbers should also help in probing the chiral invariance at short distances and SU(3) structure of electromagnetic current operator. Besides the knowledge of these c-numbers is important in itself for it would help in speculations concerning the structure of hadrons.

The various relationships, we have obtained in this chapter could be put to test once the detailed results from the $e^+ e^-$ colliding beam experiments on semi inclusive production become available.

CHAPTER IV

LIGHT CONE SINGULARITIES IN THE HIGH ENERGY BACKWARD SCATTERING

In recent years high energy scattering at large angles has received much less attention compared to diffractive processes. Earlier attempts in this direction were mainly based on the Wu and Yang's suggestion[33] that such a scattering should be proportional to the structure functions of the interacting hadrons. Available experimental data does not support this expectation. Perturbative approach was also used to study asymptotic behavior at large angles of different types of ladder graphs in various field theoretical models[34]. These diagrams were found to possess different asymptotic behaviours. Nevertheless, they exhibit a common feature. The scattering amplitude in all of them assumes a form consisting of a function which depends only on the ratio t/s of the two kinematical variables times a certain power of s. This indicates that the cross section itself may not be a function only of the ratio t/s but a quantity like $s^n \, d\sigma/dt$ might show scaling. Present experiments seem to agree with this result.

Gunion, Brodsky and Blankenbecler[35] recently obtained such a behavior of hadronic processes in a simple version of the parton model which envisages a hadron as a bound state of a point like parton and a core representing the combined effect of remaining constituents. In this picture hadrons interact via a parton interchange during high energy collisions involving large momentum transfers. Elastic scattering processes with suitable normalisation then become functions only of centre of mass scattering angle which fixes the ratio of the two large kinematic variables.

Such a behavior of strongly interacting processes very much resembles to the scaling behavior of the deep inelastic electron proton scattering which has been successfully described by various theoretical disciplines. In particular, the configuration-space analysis of the process[11] shows that the space-time separation of the two electromagnetic currents approaches the light cone region in the scaling limit and therefore the singularities of their product on the light cone play a dominating role. This observation enables one to separate out the photon energy dependence of the inelastic structure functions of the proton leaving the rest to scale.

These considerations naturally lead to the question whether light cone singularities have some role to play in the region of high energy and large momentum transfer.

It is generally understood that the singularities near the light cone give dominant contribution to processes involving a high mass current. The large mass and the large energy of the current with a fixed ratio between them lead to the light cone region. But then it is only one way to reach this region of the coordinate-space. There may exist other kinematical regions which also receive dominant contribution from the light cone region. It is this possibility which we have tried to explore in this chapter for exclusive processes[36]. In this context the following conjecture arrived at by J. Pestieau[37] by careful consideration of LSZ reduction technique is significant. At high energy strongly interacting processes are not effected by the behavior of the product of the sources inside the light cone instead appear to be dominated by the singularities on the light cone even when external masses are small. This result is suggested by the assumption of the localisation in space of the physical systems which is usually made in the quantum field theory.

We begin, in the next section, with some kinematical considerations for the pion nucleon elastic scattering

process. In Section 3, the kinematical region of large energy and momentum transfer is shown to receive the dominant contribution from the light cone region. We use the light cone expansion of the operator products to obtain the asymptotic behavior of the two amplitudes. We conclude with the discussion in the last section.

4.2 KINEMATICS:

Let us consider the elastic scattering of an iso-scalar pion off the proton,

$$\pi(q) + P(p) \rightarrow \pi(q') + P(p') \tag{4.1}$$

The S-matrix element for this process can be written as,

$$S_{fi} = \delta_{fi} - i(2\pi)^4 \, \delta(p' + q' - p - q) \, T_{fi} \tag{4.2a}$$

with

$$T_{fi} = \sqrt{\frac{M^2}{4\,\omega_q \,\omega_{q'}\, EE'}} \; \bar{u}(p') \, M(s,t) \, u(p) \tag{4.2b}$$

Here ω_q and $\omega_{q'}$ are the initial and final meson energies and E and E' the initial and final proton energies. M is the mass of the proton and the Mandelstam variables s and t are defined as follows

$$s = (p + q)^2 ; \qquad t = (q - q')^2$$

The matrix M possesses the following standard decomposition,

$$M(s, t) = - A(s, t) + \gamma \cdot Q\, B(s, t) \tag{4.2c}$$

where γ represents the Dirac matrix and $Q = \frac{1}{2}(q + q')$. Using the LSZ reduction technique, we can express the T-matrix element as,

$$T_{fi} = \frac{1}{\sqrt{4\omega_q \omega_{q'}}} \int dx\, e^{iQ.x}\, \theta(-x_o)$$

$$\langle p' | [J_\pi(-x/2), J_\pi(x/2)] | p \rangle + \text{ETC} \tag{4.3}$$

Here J_π is the pion source current defined as

$$(\Box + \mu^2)\, \phi(x) = J_\pi(x)$$

The abbreviation ETC in Eq. (4.3) denotes the equal time commutator $[J_\pi(x), \partial_o \phi(y)]$ term which is generally present when the reduction formula is written in terms of sources and depends on the details of the dynamics. If canonical commutation relations are assumed between fields this term vanishes for the pseudoscalar interaction whereas delta function singularity will arise when the source current also depends on the pion field. We, therefore, assume it contains utmost a delta function singularity. If the associated form factor vanishes sufficiently rapidly its contribution can be neglected in the asymptotic region of interest here. This is assumed to be the case.

Physically, the above limit corresponds to the high energy and large momentum transfer scattering as is easily seen by expressing Q^2 and $Q.p$ in terms of the Mandelstam variables s and t:

$$Q^2 = \frac{1}{4}(2\mu^2 + 2q.q') \approx -t/4$$

$$Q.p = \frac{1}{2}(q.p + q'.p) \approx \frac{1}{4}(2s + t)$$

Therefore,

$$Z = Q^2/Q.p = \tau/(2-\tau) \quad \text{where} \quad \tau = -t/s$$

Hence the above limit is the limit $s \to \infty$, $-t \to \infty$ with τ finite. τ is a function of the scattering angle. In the centre of mass system at high energy where the masses of the particles can be neglected it is given by,

$$\tau = \sin^2 \theta_{c.m.}/2 \tag{4.4}$$

whereas in terms of the laboratory scattering angle θ_1 it is,

$$\tau = 1 - M^2 / s \sin^2 \theta_1/2 \tag{4.5}$$

Thus Z which fixes the ratio of the two large quantities is a function of the scattering angle. This should be contrasted with the scaling variable ω for the deep inelastic electro-production process which measures the fraction of the proton momentum carried by parton.

In the centre of mass system of the proton and pion we have,

$$Q \quad = \quad (\sqrt{s}/2,\ o,\ o,(\sqrt{s}/2)(1-2\ \sin^2\ (\theta_{c.m.}/4)))$$

The exponential factor in Eq. (4.3) then becomes,

$$e^{iQ.x} \quad \approx \quad e^{i\sqrt{s}(x_o-x_3)/2\ +\ i\sqrt{s}\ \sin^2(\theta_{c.m.}/4)x_3}$$

Hence, the integral would receive major contribution from the coordinate space region satisfying,

$$|x_o - x_3| \quad \lesssim \quad 2/\sqrt{s}$$

$$x_o,\ x_3 \quad \lesssim \quad 1/\sqrt{s}\ \sin^2(\theta_{c.m.}/4)$$

Thus, in contrast to the laboratory system, for finite centre of mass scattering angle $\theta_{c.m.}$ $x \rightarrow o$ as $s \rightarrow \infty$. However, the comparison of equations (4.4) and (4.5) shows that the finite laboratory angles are mapped into the backward cone in the centre of mass system. Therefore, we can expect the singularities of the product of currents on the light cone to dominate the backward centre of mass scattering at high energy, while the short distance behavior governs the finite angle scattering.

One can now use the idea of light cone expansion of the product of two local operators together with scale invariance to express the behavior of the commutator of the pion

source currents near the light cone. The standard decomposition Eq. (4.2c) of the amplitude suggests such an expansion satisfying Lorentz invariance and the conservation of scale dimension can be written as follows:

$$[J_\pi(x/2),\ J_\pi(-x/2)] \hat{=} (\Delta(x^2))^{(d_1-2d)/2}\ O_1(x/2,\ -x/2)$$
$$+ \gamma^\mu\ \partial_\mu\ (\Delta(x^2))^{(1+d_2-2d)/2}\ O_2(x/2,-x/2) \tag{4.6}$$

where the dominant bilocal operators O_1 and O_2 correspond to the amplitudes A and B and possess finite matrix elements on the light cone $x^2 = 0$. d, d_1 and d_2 are the scale dimensions of the source operator J_π and the bilocal operators O_1 and O_2 respectively. The c-number generalised function $\Delta(x^2)$, which contains all the singularities of the commutator on the light cone, is given by,

$$(\Delta(x^2))^a = [(-x^2 + i\epsilon x_o)^a - (-x^2 - i\epsilon x_o)^a]$$

Substituting (4.6) in (4.3) we get,

$$T_{fi} = \sqrt{\frac{M^2}{4\omega_q\omega_{q'}\ E E'}}\ \bar{u}(p')\int dx\ e^{iQ.x}\ \theta(-x_o)$$
$$\{-(\Delta(x^2))^{(d_1-2d)/2}\ \tilde{A}(x^2,\ x.\Delta,\ x.P,\ t)$$
$$+ (\Delta(x^2))^{(1+d_2-2d)/2}\ \gamma\cdot Q\ \tilde{B}(x^2,x.\Delta,x.P,t)\}\ u(p) \tag{4.7}$$

The matrices $\tilde{A}(x^2, x.\Delta, x.P, t)$ and $\tilde{B}(x^2, x.\Delta, x.P, t)$ are defined as follows:

$$< p'|O_1(x/2, -x/2)|p > \; = \; - \sqrt{\frac{M^2}{EE'}} \, \bar{u}(p') \, \tilde{A}(x^2, x.\Delta, x.P, t) u(p) \qquad (4.8)$$

$$< p'|\gamma^\mu \, O_2(x/2, -x/2)|p > \; = \; \sqrt{\frac{M^2}{EE'}} \, \bar{u}(p') \gamma^\mu \, \tilde{B}(x^2, x.\Delta, x.P, t) u(p) \qquad (4.9)$$

where $P = \frac{1}{2}(p+p')$ and $\Delta = p'-p = q-q'$.

From equations (4.2b) and (4.7) we obtain,

$$A(s,t) = \int dx \, e^{iQ.x} \, \theta(-x_o) \, (\Delta(x^2))^{(d_1-2d)/2} \, \tilde{A}(o, x.\Delta, x.P, t) \qquad (4.10a)$$

and

$$B(s,t) = \int dx \, e^{iQ.x} \, \theta(-x_o) \, (\Delta(x^2))^{(1+d_2-2d)/2} \, \tilde{B}(o, x.\Delta, x.P, t) \qquad (4.10b)$$

Now to carry out the x-integration we define the double Fourier transform $g_1(\alpha, \beta, t)$ of $\tilde{A}$ with respect to the variables $x.\Delta$ and $x.P$

$$\tilde{A}(x.\Delta, x.P, t) = \int \int d\alpha \, d\beta \, g_1(\alpha, \beta, t) \, e^{i(\alpha x.\Delta + \beta x.P)} \qquad (4.11)$$

The variables α and β will, in general, have infinite range and therefore Eq. (4.11) may not exist. Its validity essentially requires the existence of Deser-Gilbert-Sudarshan type representation for non-forward elastic scattering

amplitudes. Such a representation was first written by Nakanishi[38]. Using spectral conditions the function $g_1(\alpha,\beta,t)$ has been found to have finite support in the region $|\beta| \leq 1$. We further hope that only finite values of α contribute to Eq. (4.11). Making use of the identity,

$$\int dx\, e^{i\xi . x}\, \theta(-x_o)\, (\Delta(x^2))^a \equiv c(a)\, h(\xi^2, \xi_o) \qquad (4.12)$$

with $c(a) = \pi^{2a+3}\, \Gamma(a+1)\, \Gamma(a+2)$,

$$h(\xi^2, \xi_o) = \exp\,(-i\pi\epsilon(\xi_o)[(\Delta(\xi^2))^{-a-2} + (\Delta(-\xi^2))^{-a-2}]$$

Equations (4.10) become,

$$A(s,t) = c(d_1-2d)/2) \int\int d\alpha\, d\beta\, g_1(\alpha, \beta, t)\, h_1(\xi^2, \xi_o) \qquad (4.13a)$$

$$B(s,t) = c(1+d_2-2d)/2) \int\int d\alpha\, d\beta\, g_2(\alpha, \beta, t)\, h_2(\xi^2, \xi_o) \qquad (4.13b)$$

where,

$$\xi^2 = (Q + \alpha\Delta + \beta P)^2 \approx s\, \delta(\alpha, \beta, \tau)$$

$$\xi_o = (Q + \alpha\Delta + \beta P)_o \approx \sqrt{s}\, \delta'(\beta)$$

for large s and t. Finally with the assumption that asymptotic behavior of the amplitude is determined solely by the dominant singularity on the light cone, we obtain,

$$A(s,t) \approx s^{-(d_1-2d)/2\,-2}\, f_1(t/s) \qquad (4.14a)$$

and $$B(s,t) \approx s^{-(1+d_2-2d)/2\,-2}\, f_2(t/s) \qquad (4.14b)$$

with,

$$f_i(t/s) \;=\; \iint d\alpha\, d\beta\, g_i\, (\alpha,\, \beta,\, t)\; h_i\, (\delta,\, \delta'/\sqrt{s})$$

Such a behavior of the amplitude should be compared with its Regge behavior $s^{\alpha(t)}\ \beta(t)$ at small momentum transfers. If there exists a function which connects the two regions there should be a relation between the trajectory function and the dimensions of the operators - the nature of which is not clear at present.

4.4 DISCUSSION:

Now we can determine the high energy and large momentum transfer limit of the differential cross-section of the reaction (4.1). The standard procedure gives,

$$\frac{d\sigma}{dt} \simeq \frac{1}{16\pi\ s^2}\,[\tau s|A|^2 + s^2(1-\tau)|B|^2 - s\ M(2-\tau)(AB^* + BA^*)] \quad (4.15)$$

using (4.14) we obtain,

$$\begin{aligned}
\frac{d\sigma}{dt} &\simeq s^{2d-5-d'}\ f(\tau) && \text{for } d_1 = d_2 = d' \\
&\simeq s^{2d-5-d_1}\ f'(\tau) && \text{for } d_1 < d_2 \\
&\simeq s^{2d-5-d_2}\ f''(\tau) && \text{for } d_2 < d_1
\end{aligned}$$

This is as far as light cone ideas can lead us. In this approach the scale dimensions d_1, d_2 and d remain unspeccified. They can, in general, be different from the physical

dimensions and may even depend on the interaction parameter. To supply this missing information we turn towards experiments.

Chikashiga and Inagaki[39] have studied high energy photoproduction of pseudoscalar meson using essentially the same technique as ours. Using the forward differential cross section data they obtain the value $d = 2$ for the scale dimension of the pion source current. We take this value for d. In Fig. 4, we have plotted the available experimental data[40] for $\pi^- p$ cross section in the near backward direction against the energy variable s. The slope of the straight line obtained shows that the differential cross section $d\sigma/dt$ falls as s^{-3}. Combining this experimental information with the value $d = 2$ we get,

$$d_1 = d_2 = 2$$

Thus, twist two bilocal operators seem to dominate the singularity structure of the commutator of the two pion source currents near the light cone. It should be recalled here that the light cone behavior of the electromagnetic current commutators is also dominated by the twist two operators.

If the pion source current dimension is assumed to be $d = 3$ following quark model, then d_1 $(= d_2) = 4$; the field operator that contributes in the operator product expansion has at least a dimension of 4.

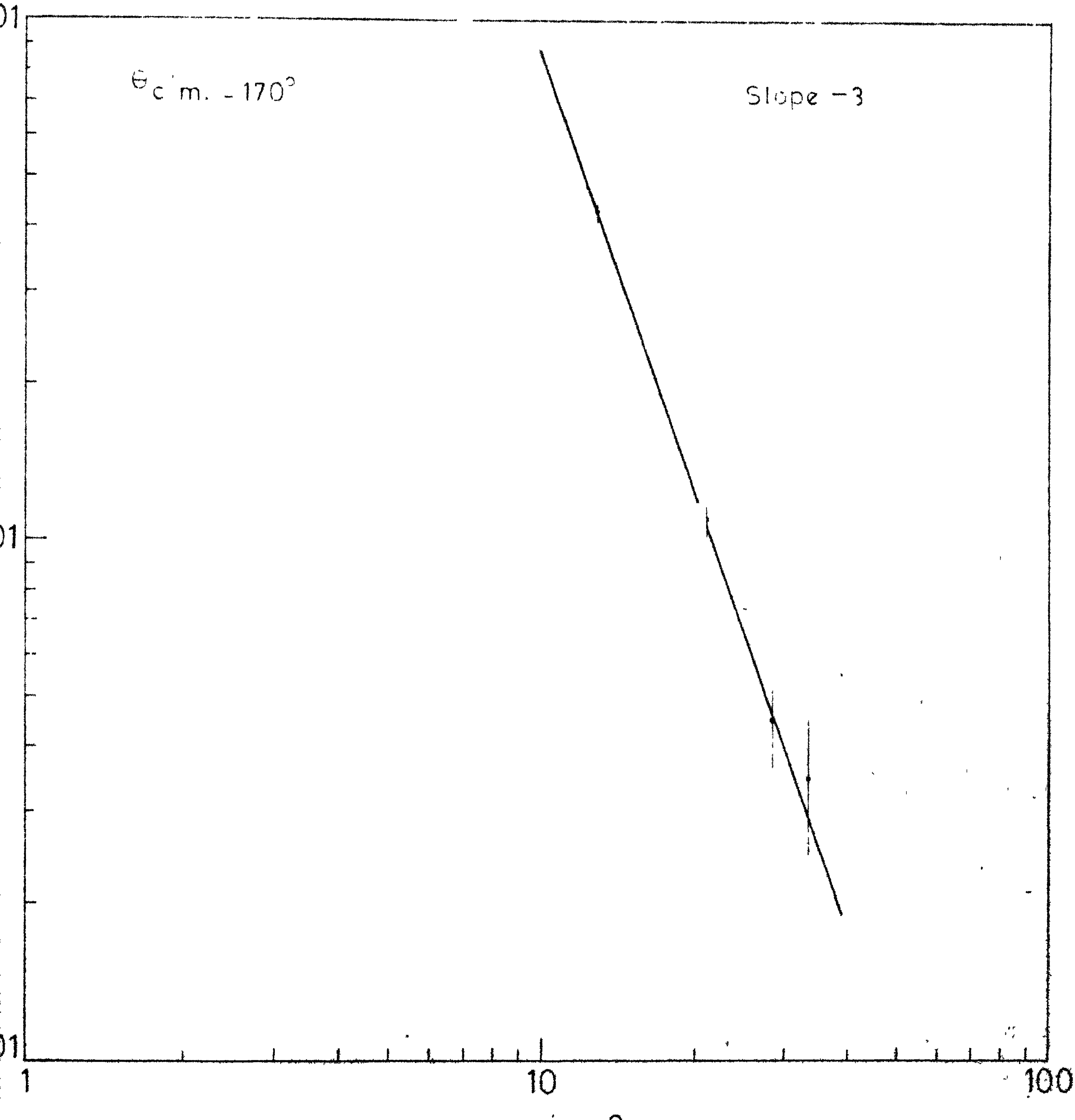
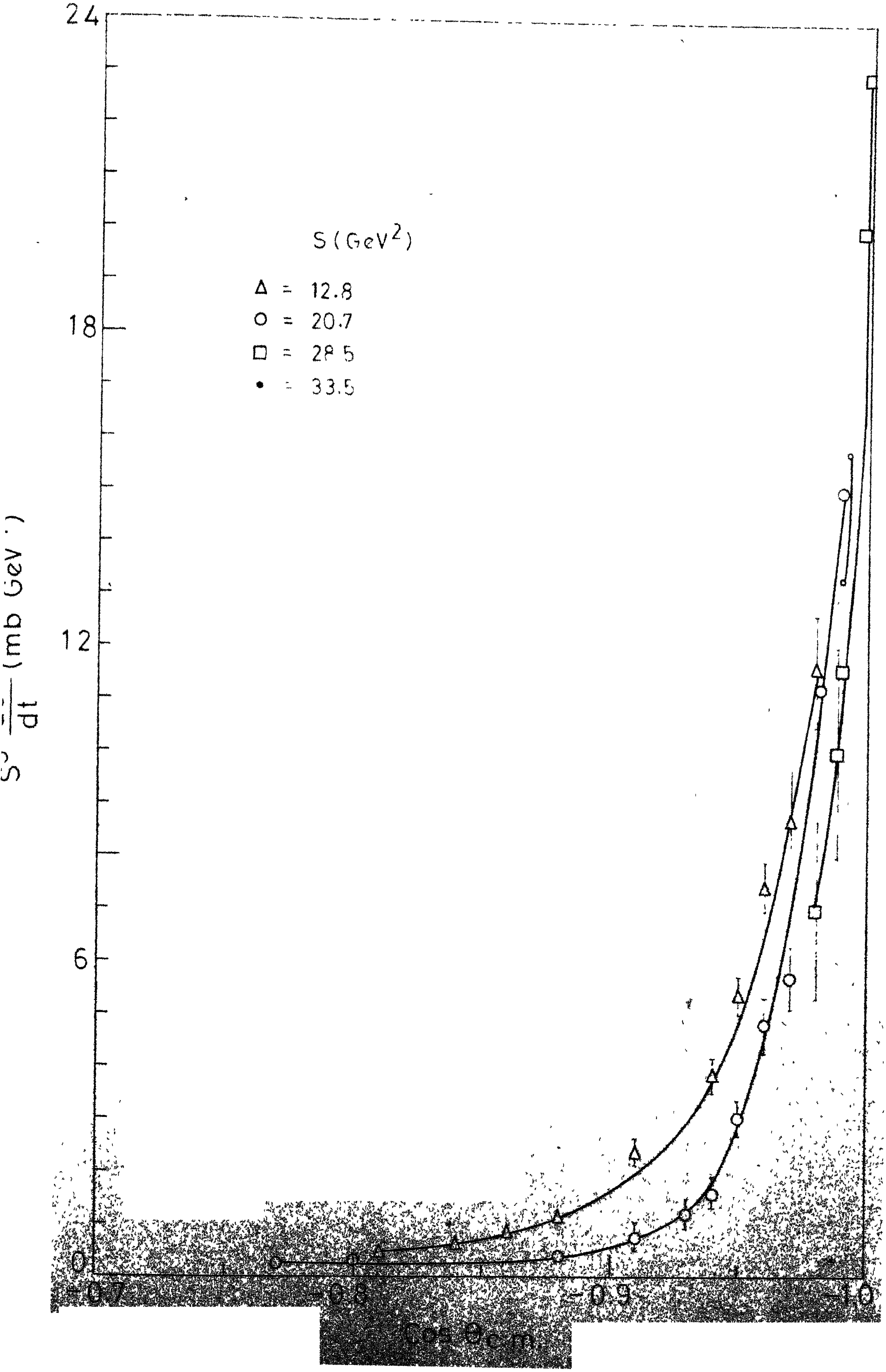


Fig. 4

Now it has been observed that at $\theta_{c.m.} = 90^{\circ}$ the differential cross section for $\pi^{-}p$ falls as s^{-8}. This rapid decrease has been comprehended on the basis of a single parton interchange between the interacting hadrons. In such a model, the differential cross section becomes proportional to the square of the form factors of the hadrons. This implies its sharp fall with energy as these form factors usually decrease in momentum transfer like a dipole. The backward π-p elastic scattering in the centre of mass system which shows a s^{-3} decrease is difficult to understand on such a basis. On the other hand it indicates the dominant role of the singularities on the light cone of the product of the pion sources. The precise relation between this parton model presumably applicable for finite c.m. angles and the light cone approach used here for the backward cone requires to be established.

Figure 5 is an attempt to obtain the scaling function $f(\tau)$ which depends on the details of the dynamics. The plot for $s^{3}\,\frac{d\sigma}{dt}$ versus $\cos\theta_{c.m.}$ for the four energies does indeed have a tendency to fuse together for nearly backward angles and is in accordance with our expectation. However, the data is not yet capable of giving unambiguously the scaling function. Indeed, outside the backward cone, the curves for various energies are not quite overlapping but our hope



24
18
12
6
0
S(GeV^2)
Δ = 12.8
O = 20.7
□ = 28.5
• = 33.5
-0.7
-0.8
-0.9
-1.0
Cos θc.m.

is that with increase in energy tendency for merger should show up in this region also.

In conclusion, the backward region in the $\pi^{-}p$ elastic scattering appears to receive dominant contribution from the light cone region of the coordinate space. There is, however, a need to obtain a more accurate data at large angles for fairly large high energy range so that the energy dependence may be found with certainty. This should also establish clearly the region where light cone effects become important.

CHAPTER V

INCLUSIVE HADRONIC REACTIONS AND LIGHT CONE

The study of single particle inclusive spectrum in high energy two particle collision processes has been the subject of considerable theoretical and experimental interest. Such a study was stimulated mainly by the two well known scaling laws. One is the scaling behavior of the deep inelastic lepton-hadron scattering suggested by Bjorken[41]. The second one is the limiting behavior at high energies of the single particle distributions in hadronic inclusive reactions conjectured by Feynman and Benecke et.al.[42] These laws have been discussed in a number of models, namely, dual resonance model, parton model,light cone algebra model etc. Of late it was shown within the framework of the dual resonance model[43] that both these laws possess the same physical origin - the Regge behavior in various channels together with a Regge pole of intercept unity. Now the Bjorken scaling of the inelastic structure functionsof proton has also been studied from the point of view of coordinate space[11]. It turns out that the observed scaling is due to the dominance of the leading light cone singularity of the product of two electromagnetic currents. Therefore

it seems reasonable to inquire about the relevance, if any, of the light cone singularities in the Feynman scaling region of inclusive reactions where the single particle distribution possesses simple forms.

Such a relevance has been shown for the electroproduction process where one of the produced hadrons is also observed. A generalised Feynman scaling, where the distribution function at high energies depends on both the usual Feynman variable x_F and the Bjorken scaling variable, has been obtained[44]. Since in this reaction the scaling sets in at unexpectedly low values of the virtual photon mass one may expect prominent role of the light cone region in hadronically induced reactions.

In this chapter we show that light cone singularities could play an important role in the fragmentation and pionization regions. The standard forms of the distribution function are obtained if we may expect the matrix elements of multilocal operator to avoid a certain singular behavior.

5.2 THE REACTION $\pi + N \to \pi$ + ANYTHING:

Let us consider, to be specific, the following inelastic process,

$$\pi(q) + N(p) \to \pi(q') + X \text{ (anything)} \tag{5.1}$$

Here N stands for a nucleon and the isospin labels are not displayed, since they are not of any relevance for the dynamics here. The differential cross section for such a process can be expressed in the following form,

$$\frac{d\sigma}{d^3q'} = \frac{(2\pi)^3 M}{4E\ \omega_q \omega_{q'} V} \int dx\ e^{iq.x} \sum_X \langle p|J_\pi(x)|\pi\ X\rangle\langle \pi X|J_\pi(o)|p\rangle \tag{5.2}$$

where J_π is pion source current operator and an average over the polarisation of the initial nucleon is implied. ω_q and $\omega_{q'}$ are the initial and final meson energies and E the energy of target nucleon. M is the nucleon mass and V the relative velocity of the two colliding particles.

Using Lehman - Symanzik -Zimmerman reduction technique for the pion in the first matrix element in Eq.(5.2) and then making change of integration variables, the integral I in Eq. (5.2) becomes,

$$I = \frac{i}{\sqrt{(2\pi)^3}} \int dz\ e^{iQ.z} \sum_X (2\pi)^4 \langle p|RJ_\pi(Z/2)\ J_\pi(-Z/2)|X\rangle$$
$$\langle \pi X|J_\pi(o)|p\rangle\ \delta(p_X + q' - p - q)$$

$$= \frac{i}{\sqrt{(2\pi)^3}} \int dz\ dx\ e^{iQ.z}\ e^{-iq.x} \sum_X \langle p|RJ_\pi(z/2)J_\pi(-z/2)|X\rangle$$
$$\langle \pi X|J_\pi(x)|p\rangle$$

where $Q = \frac{1}{2}(q + q')$. Again reducing in the pion from

second matrix element we get,

$$I = \frac{i^2}{(2\pi)^3} \int dz\, e^{iQ.z} \int dx\, dy\, e^{-iq.x - iq'.y}$$

$$\sum_X \langle p| RJ_\pi(z/2)\, J_\pi(-z/2)|\, X \rangle\langle X| RJ_\pi(x) J_\pi(y)|p \rangle$$

Equation (5.2) now assumes the following form

$$\frac{d\sigma}{d^3q'} = \frac{(2\pi)^3 M}{4E\,\omega_q \omega_{q'} V} \int dz\, e^{iQ.z}\, f(z;p,q,q') \tag{5.3}$$

The function f(z; p,q,q') containing the interesting dynamics is given by,

$$f(z;\ p,\ q,q') = \frac{i^2}{(2\pi)^3} \int dx\, dy\, e^{iq'.y-iq.x}$$

$$\langle\, p| R(J_\pi(z/2)J_\pi(z/2))R(J_\pi(x)J_\pi(y))|p \,\rangle \tag{5.4}$$

and

$$R(J_\pi(x)J_\pi(y)) = \theta(x_o-y_o)\ [J_\pi(x),\ J_\pi(y)]$$

denotes the retarded commutator. The terms which appear when reduction formula is written in terms of sources have been omitted for the reasons given in the last chapter.

Now let us examine the behavior of Eq. (5.3) in the limit $Q^2 \to \infty$ and $Q.p \to \infty$ with a fixed ratio κ between

them. It is convenient to work in the rest frame of the target nucleon ($p_o = M$) where

$$Q^2/Q.p = Q^2/Q_oM = \kappa$$

and therefore,

$$|\bar{Q}| = Q_o \, [1 - Q^2/Q_o^2 \,]^{1/2}$$

$$\approx Q_o - M\kappa/2$$

Choosing the z_3-axis along the $\bar{Q}$-vector the exponential in Eq. (5.3) becomes,

$$e^{iQ.z} \approx e^{iQ_o(z_o-z_3)+i(M\kappa/2)z_3}$$

Using the arguments given in the Section 4.3 we see that the integral in **Eq.** (5.3) receives dominant contribution from the region of coordinate-space satisfying $z^2 \approx o$. Thus, the above limit is governed by the singularity structure of the function $f(z;\, p,q,q')$ near the light cone.

Equation (5.4) shows that the structure of $f(z;\, p,\, q,\, q')$ near $z^2 \approx o$ is governed by that of the following operator product

$$R(J_\pi(z/2)J_\pi(-z/2))\ R\ (J_\pi(x)J_\pi(y)) \qquad (5.5)$$

Here the two current operators whose space time separation approaches light cone are in the same R-product in contrast to virtual photon inclusive reactions. Therefore no

additional assumption[45] is needed to conclude that the function $f(z; p, q, q')$ has the same singularity structure as the product of the two pion sources. Now J.Ellis[46] has suggested an extension of the light cone expansion of two operator product to that of the multiple operator product to discuss the scaling behavior of inclusive electromagnetic processes. Following him we write,

$$R(J_\pi(z/2)J_\pi(-z/2))R(J_\pi(x)J_\pi(y))$$

$$\underset{z^2\to o}{\equiv} \theta(z_o)(\Delta(z^2))^d \sum_n z^{\alpha_1}\ldots z^{\alpha_n} O^{(n)}_{\alpha_1\ldots\alpha_n}(x,y,o) \tag{5.6}$$

where $O^{(n)}_{\alpha_1\ldots\alpha_n}(x,y,o)$ are multilocal operators and again usual c-number function $\Delta(z^2)$ contains all the singularities on the light cone $z^2 \approx o$. The strength d of the singularity depends on the scale dimensions of J_π and the multilocal operators. However, as seen in the last chapter, the information about d may also be obtained from the high energy backward elastic scattering of pion off the proton. The experimental data indicates the value -1 for d, whereas the free quark model gives $d = -2$.

Substituting Eq.(5.6) into Eq. (5.4) we get,

$$f(z;\ p,q,q') \approx \frac{\theta(z_o)(\Delta(z^2))^d}{(2\pi)^3} \int dx\, dy\, e^{iq'.y-iq.x}\, \tilde{g}(z,x,y;p)$$

$$\approx \frac{\theta(z_o)(\Delta(z^2))^d}{(2\pi)^3} \tilde{f}\,(z.p,\ z.q, z.q';\ s,t_1,t_2) \tag{5.7}$$

This relation displays explicitly the singularity in z^2. The function $\tilde{g}(z, x,y;p)$ defined as,

$$< p|-\sum_n z^{\alpha_1}\ldots z^{\alpha_n} O^{(n)}_{\alpha_1\ldots\alpha_n} (x,y,o)|p > \;=\; \tilde{g}(z,x,y;\, p) + \ldots$$

is independent of z^2 but otherwise depends on all possible scalars which can be obtained from x, y, z and p. The omitted terms contribute to lower order light cone singularity. s, t_1 and t_2 are the Mandelstam variables defined by,

$$s = (p + q)^2$$
$$t_1 = (p - q')^2$$
$$t_2 = (q - q')^2$$

From Eqs. (5.7) and (5.3) we obtain,

$$\omega_{q'} \frac{d\sigma}{d^3q'} = \frac{M}{4E\,\omega_q V} \int dz\, e^{iQ.z}\, \theta(z_o)(\Delta(z^2))^d$$
$$\tilde{f}(z.p, z.q, z.q';\, s, t_1, t_2) \qquad (5.8)$$

To carry out the z-integration we define Fourier transform $g(\alpha, \beta, \beta';\, s,t_1, t_2)$ of $\tilde{f}$ with respect to the variables z.p, z.q and z.q':

$$\tilde{f}\,(z.p,\, z.q,\, z.q';\, s,\, t_1,\, t_2) =$$
$$= \int_{-\infty}^{\infty} d\alpha\, d\beta\, d\beta'\, e^{i(\alpha p+\beta q+\beta' q').z}\, g(\alpha,\beta,\beta';\, s,t_1,t_2) \quad (5.9)$$

The limits on α, β and β' are formal. The actual support of g is determined by the mass spectrum which we hope is finite.

Substituting (5.9) in (5.8) and using Eq. (4.12) we obtain,

$$\omega_{q'} \frac{d\sigma}{d^3q'} = \frac{MC(d)}{4E\,\omega_q V} \int d\alpha\, d\beta\, d\beta'\; g(\alpha,\beta,\beta';\; s,t_1,\; t_2)\, h(K^2,K_o) \tag{5.10}$$

where, $K = (Q + \alpha p + \beta q + \beta' q')$ and

$$h(K^2,K_o) = [(\Delta(K^2))^{-d-2} + (\Delta(K^2))^{-d-2}]\, \exp(-i\pi\mathcal{E}(K_o))$$

5.2.1 Fragmentation Region:

Now let us examine the limit $Q^2 \to \infty$, $Q.p \to \infty$ with the ratio $Q^2/Q.p$ fixed. For this purpose we express both the invariants in terms of s, t_1 and t_2:

$$Q^2 = \frac{1}{4}(2\mu^2 + 2q.q') \approx -t_2/4$$

and

$$Q.p = \frac{1}{2}(q.p + q'.p) \approx \frac{1}{4}(s - t_1)$$

Therefore,

$$\kappa = \frac{Q^2}{Q.p} = \frac{4\mu^2/s - t_2/s}{(1-t_1/s)} \approx \frac{-t_2/s}{1-t_1/s}$$

Thus the above limit corresponds to the region of high s with t_1 or t_2 as its finite fraction which is the fragmentation region. For definiteness we keep t_1 finite and allow s and t_2 to go to infinity with $-t_2/s = \kappa'$ fixed. κ coincides with κ' in this case and we get, in general[47]

$$K^2 = (Q + \alpha p + \beta q + \beta' q')^2 \simeq s\left[\frac{\kappa'}{4} + \frac{\alpha}{2} + \beta\beta'\kappa' + \alpha\beta + \frac{(\beta+\beta')}{2}\kappa'\right.$$

$$\simeq s\,\delta(\alpha, \beta, \beta'; \kappa') \tag{5.11}$$

$$K_0 = (Q + \alpha p + \beta q + \beta' q')_0 \simeq \sqrt{s}\,\delta'(\alpha, \beta, \beta') + O(M_X^2/\sqrt{s})$$

in the centre of mass system. Now the function $g(\alpha, \beta, \beta'; s, t_1, t_2)$ in Eq. (5.10), which is related to the matrix element of the multilocal operator, depends on the details of strong interaction dynamics and is therefore impossible to calculate. In such a situation we assume it has the following Regge-like behavior:

$$\lim_{\substack{s \to \infty \\ -t_2 \to \infty}} g(\alpha, \beta, \beta'; s, t_2, t_1) \to s^n g_1(\alpha, \beta, \beta'; \kappa', t_1) \tag{5.12}$$

Using (5.11) and (5.12) we obtain,

$$\omega_{q'} \frac{d\sigma}{d^3q'} \sim \frac{1}{s^{d+3-n}} \int d\alpha\, d\beta\, d\beta'\, h(\delta, \delta'/\sqrt{s})\, g_1(\alpha, \beta, \beta'; \kappa', t_1)$$

$$\sim \frac{1}{s^{d+3-n}} f_1(\kappa', t_1) \tag{5.13}$$

where we have assumed the convergence of the integral in Eq. (5.13), which in turn precludes any singularities[48] of g_1 on the real axis of α, β and β'. Equation (5.13) resembles very much the Feynman's scaling result for the single particle distribution. To see it more explicitly we notice,

$$\begin{aligned} s &= 4\, p^2_{c.m.} \\ t_2 &= 2\mu^2 - \sqrt{s}(q'_o + q'_{11}) \\ t_1 &= M^2 + \mu^2 - \sqrt{s}\,(q'_o - q'_{11}) \end{aligned} \tag{5.14}$$

in the centre of mass frame. Now for fragment pions moving with high longitudinal momentum q'_{11} we can rewrite these in terms of the Feynman variable $x_F = 2q'_{11}/\sqrt{s}$. For $x_F > o$, (q'_{11} has same sign as $\vec{p}$)

$$t_1 = \mu^2 - \frac{\mu_T^2}{x_F} + M^2\,(1-x_F); \quad \mu_T^2 = \mu^2 + q_T'^2$$

$$t_2 = -sx_F$$

Thus

$$\kappa' = -t_2/s = x_F$$

For $x_F < o$,

$$t_2 = \frac{\mu_T^2}{x_F} + \mu^2\,(2 + x_F)$$

$$t_1 = s\,x_F\,; \quad t_1/s = x_F$$

This corresponds to the case of fixed t_2 and large s, t_1 which is the projectile fragmentation region.

Thus, in the fragmentation region, κ' coincides with the Feynman variable x_F as it should. In this region Mueller's Regge hypothesis gives,

$$\omega_{q'}\,\frac{d\sigma}{d^3q'} \approx \frac{1}{s^{1-\alpha(t)}}\, f_2(x_F,\, t_1)$$

Comparison with Eq. (5.13) shows a relation between the strength of the singularity d and the Regge trajectory function $\alpha(t)$. An analogous result, in the case of exclusive process was obtained by M. Eishari[49] and favours non-canonical dimensions. With pomeron dominance, for Feynman kind of scaling n must have the value given by,

$$\begin{aligned} n &= d + 3 \\ &= 1 \qquad\qquad \text{for } d = -2 \\ &= 2 \qquad\qquad \phantom{\text{for }} d = -1 \end{aligned}$$

5.2.2 Pionization Region:

The pionization region is defined by the limit $s, -t_1, -t_2 \to \infty$ such that the ratio $t_1 t_2/s$ remains fixed. This region is characterised by slow moving pions i.e. q' is finite. We express t_1 and t_2 in terms of commonly used variables s, μ_T and the rapidity y:

$$t_1 \approx - \sqrt{s}\,(q_0' - q_{11}') \approx - \sqrt{s}\,\mu_T\, e^{-y}$$

$$t_2 \approx - \sqrt{s}\,(q_0' + q_{11}') \approx - \sqrt{s}\,\mu_T\, e^{y}$$

The rapidity y of the final pion is defined as,

$$y = \frac{1}{2} \log \frac{q_0' + q_{11}'}{q_0' - q_{11}'} \approx \frac{1}{2} \log (t_2/t_1)$$

and $\mu_T^2 = q_T'^2 + \mu^2$. Now in this region

$$Q_o = \frac{1}{2}(q_o + q_o') \approx \frac{q_o}{2} ; \quad |\bar{Q}| \approx \frac{q_o}{2} + \frac{q_{11}'}{2}$$

Therefore writing the exponential in Eq. (5.3) as
$\exp i\,[\frac{1}{2}(Q_o + |\bar{Q}|)(z_o - z_3) + \frac{1}{2}(Q_o - |\bar{Q}|)(z_o + z_3)]$
it is easily seen that only the region $|z_o - z_3| < \frac{1}{q_o}$, $|z_o + z_3| < \frac{1}{q_{11}'}$ i.e. $z^2 \approx o$ contributes to the integral as $q_o \to \infty$.

In this case K^2 and K_o in Eq. (5.10) become,

$$K^2 \approx s(\frac{\alpha}{2} + \alpha\beta) + \sqrt{s}\,(\frac{1}{4} + \frac{\beta+\beta'}{2} + \beta\beta')\;\; \mu_T\, e^{y}$$

$$+ \sqrt{s}\,(\frac{\alpha}{2} + \alpha\beta')\;\mu_T\, e^{-y}$$

$$K_o \approx \sqrt{s}\,(\frac{1}{4} + \frac{\alpha+\beta}{2})$$

Hence the function $h(K^2, K_o)$ becomes independent of the rapidity y in the limit $s \to \infty$. Consequently a flat rapidity distribution in the central region is ensured if we assume that all the essential details are contained in the singularity structure. With a Regge like behavior for $g(\alpha, \beta, \beta'; s, t_1, t_2)$ in this Kinematical region as well, similar to Eq. (5.12).

$$\lim_{s,-t_1,-t_2 \to \infty} g(\alpha,\beta,\beta'; s,t_1,t_2) = s^n\, g_2(\alpha,\beta,\beta'; \mu_T^2, y)$$

with

$$t_1 t_2/s = \mu_T^2 \quad \text{fixed}$$

$$\log\; t_2/t_1 = y \quad \text{fixed}$$

we then have, analogous to Eq. (5.13),

$$\omega_{q'} \frac{d\sigma}{d^3q'} \approx \frac{1}{s^{d+3-n}} \int d\alpha\, d\beta\, d\beta'\, h\,(\delta,\, \delta'/\sqrt{s})\; g_2(\alpha,\beta,\beta';\, \mu_T^2,\, y)$$

where δ and δ' are functions of only α, β and β'. The y dependence of the inclusive spectrum arises, if at all, from the behavior of g_2 and not of the singularity function. We then have,

$$\omega_{q'} \frac{d\sigma}{d^3q'} \approx \frac{1}{s^{d+3-n}}\; F(\mu_T^2,\, y)$$

and will conform to the known behavior of single particle distribution, in the central region if as in fragmentation region $n = d + 3$ and the rapidity dependence of the g-function is weak. We may then write,

$$\omega_{q'} \frac{d\sigma}{d^3q'} \approx F(\mu_T^2)$$

which is the standard form with a central plateau in the rapidity variable.

5.3 CONCLUSION:

We conclude this chapter with few comments. The above analysis shows the relevance of the light cone singularities to the limiting fragmentation and the central regions of hadronic inclusive reactions. The simple bahvior of single particle distribution in these

regions seems attributable to the t/s scaling in the framework of light cone analysis. This correspondence is our main result. It should be compared with an analogous relation of large transverse momentum $q_\perp'$ scaling in inclusive reactions to t/s and u/s scaling established in the quark and parton models[50]. So it appears that in both low $q_\perp'$ and large $q_\perp'$ inclusive reactions small distance behavior of strong interactions is involved. The y-independence of the function $h(K^2, K_o)$ in the limit $s \to \infty$, a non trivial result, ensures the flat pion spectrum in the rapidity variable. The constraint of keeping particle on the mass-shell leads to nonforward directions complicating the configuration space analysis with dynamical details not encountered in its application to electromagnetic processes. This necessitates incorporation of some dynamical assumptions. These require matrix elements of multilocal operators to possess Regge-like behaviors. At present their justification is not possible but they certainly reveal the kind of obstacles to be overcome before small distance expansion of operator products, as suggested by Wilson[11], can describe the hadronic interactions. An alternative derivation of scaling behavior shown by single particle distribution is

not claimed here. The important thing is the connection of light cone region of coordinate space to Feynman's scaling. In this context the above analysis appears useful as it suggests the possibility of the same origin for Bjorken scaling and Feynman scaling. If true, this may provide a unified basis for describing electromagnetic and hadronic inclusive reactions.

CHAPTER VI

CONCLUDING REMARKS

The preceeding chapters deal with the processes involving one or more pions. The analyses of these reactions involve two currents - the source current and the isovector axial vector current associated with the pion. The efforts have been devoted to connect the short distance parameters occurring in the operator products involving these currents, to experimentally observable quantities. Thus (i) the vector anomaly of the axial vector-vector-vector vertex function, (ii) the constants R_A and R_V occurring in the operator product expansion of $A_\mu(x)\ A_\mu(o)$ and $V_\mu(x)\ V_\mu(o)$ and (iii) the scale dimensions of the operators appearing in the light cone expansion of the product $J_\pi(x)\ J_\pi(o)$, have been related to the various differential cross-sections. All these parameters characterize short distance structure of a given theory and have a bearing on the hadronic structure. The presence of the vector anamoly substantially affects the production of neutral pion in the two (virtual) photon mechanism in the $e^+ e^-$ collisions. Since there is a kinematical enhancement of such a production mechanism, as pointed out by Broadsky, it is likely that the experimental results will soon be in a position to test whether such terms are required.

Whereas, the axial vector and vector current are as usual studied using electromagnetic probes, it is interesting to speculate on the use of the properties of the currents in purely hadronic interactions. The last two chapters are devoted to the use of the parameters associated with the pion source currents in the πN interactions. Our results for differential cross-section take the form of a definite power of s times a scale invariant function of remaining invariants. The limitations of our analyses and other possibilities have also been discussed. The connection of Feynman scaling of single particle inclusive distribution to the light cone singularities shown in last chapter is rather speculative at the level of present study. However, the studies in dual resonance model suggesting the same origins for Bjorken scaling and Feynman scaling increase our faith in the result. But a great deal of work will have yet to be done before such a relationship can be firmly established.

APPENDIX A

Our metric is (1, -1, -1, -1). The covariant spin-one parts of the vector and the axial vector propagators are given by,

$$\Delta_V^{\mu\nu}(k) = \int_0^\infty d\mu^2 \frac{\rho_V(\mu^2)}{(k^2 - \mu^2)} (g^{\mu\nu} - k_\mu k_\nu / \mu^2) \qquad (A.1)$$

$$\Delta_A^{\mu\nu}(k) = \int_0^\infty d\mu^2 \frac{\rho_A(\mu^2)}{(k^2 - \mu^2)} (g^{\mu\nu} - k_\mu k_\nu / \mu^2) \qquad (A.2)$$

where the spectral functions $\rho_V(\mu^2)$ and $\rho_A(\mu^2)$ are defined as,

$$< o|V_a^\mu(x)V_b^\nu(o)|o > = \delta_{ab} (2\pi)^{-3} \int dp\ \theta(p_o)\ e^{ip.x} \rho_V(p^2) (g^{\mu\nu} - p^\mu p^\nu/p^2)$$

$$< o|A_a^\mu(x)A_b^\nu(o)|o > = \delta_{ab} (2\pi)^{-3} \int dp\ \theta(p_o)\ e^{ip.x} [\rho_A(p^2)(g^{\mu\nu}-p^\mu p^\nu/p^2)-f_\pi^2\ p^\mu p^\nu]$$

$\Delta^{\mu\nu}(k)_{\omega,\phi}$ is the usual massive vector meson propagator and

$$\Delta_\pi(k) = \frac{f_\pi^2\ m_\pi^4}{(k^2-m_\pi^2)}$$

The constants appearing in Eq. (2.12) are defined as,

The constants appearing in Eq. (2.12) are defined as,

$$(2\pi)^{3/2} \sqrt{2k_o} < o| V_a^\mu | \rho_b^\nu(k) >$$

$$= - g_\rho \; \delta_{ab} \left(g^{\mu\nu} - k^\mu k^\nu / m_\rho^2\right)$$

$$(2\pi)^{3/2} \sqrt{2k_o} < o| A_a^\mu | A_{1b}^\nu (k) >$$

$$= - g_{A_1} \delta_{ab} \left(g^{\mu\nu} - k^\mu k^\nu / m_{A_1}^2\right)$$

and

$$(2\pi)^{3/2} \sqrt{2k_o} < o| A_a^\mu | \pi_b (k) >$$

$$= i f_\pi \, \delta_{ab} \, k^\mu$$

APPENDIX B

We sketch here the derivation of equations (2.29)-(2.33) of chapter two. The coupling of a pseudoscalar meson to two vector particles is defined as,

$$T_{\nu\lambda}\ (k,\ k_1) \ = \ f_{VVP}\ \epsilon_{\nu\lambda\,\alpha\beta}\ k^{\alpha} k_1^{\beta} \tag{B.1}$$

with k and k_1 as vector particles momenta. We invert equations (2.10)-(2.11) to write,

$$\omega_\mu = \frac{1}{a_\omega}\left\{-\frac{\sin\theta_B}{f_Y'}\ V_\mu^8 + \frac{\cos\theta_Y}{f_B'}\ V_\mu^0\right\}$$

$$\phi_\mu = \frac{1}{a_\phi}\left\{\frac{\cos\theta_B}{f_Y'}\ V_\mu^8 + \frac{\sin\theta_Y}{f_B'}\ V_\mu^0\right\} \tag{B.2}$$

where

$$a_\omega = m_\omega^2 \quad \cos\left(\theta_Y - \theta_B\right),\ a_\phi = m_\phi^2 \cos\left(\theta_B - \theta_Y\right)$$

Also

$$\rho_\mu^a = \frac{1}{g_\rho}\ V_\mu^a \tag{B.3}$$

and $\quad J_\mu^{e.m.} = e(V_\mu^3 + \frac{1}{\sqrt{3}}\ V_\mu^8)$

Using LSZ reduction technique and equations (B.2) and (B.3) together with the charge conjugation invariance the amplitudes $T_{\nu\lambda}$ for the decays $\rho\to\pi\gamma$, $\omega\to\rho\pi$, $\omega\to\pi\gamma$, $\phi\to\rho\pi$ and $\phi\to\pi\gamma$ can be written as follows:

$$T_{\nu\lambda}^{\rho\to\pi\gamma} = -\frac{e(k^2 - m_\rho^2)}{\sqrt{3}\, f_\pi\, g_\rho} M(q,k)_{\nu\lambda}^{338} \qquad (B.4)$$

$$T_{\nu\lambda}^{\omega\to\pi\gamma} = -\frac{e(k^2 - m_\omega^2)}{f_\pi\, a_\omega} \left[\frac{\sin\theta_B}{f'_Y} M(q,k)_{\nu\lambda}^{338} + \frac{\cos\theta_Y}{f'_B} M(q,k)_{\nu\lambda}^{330}\right] \qquad (B.5)$$

$$T_{\nu\lambda}^{\phi\to\pi\gamma} = -\frac{e(k^2 - m_\phi^2)}{f_\pi\, a_\phi} \left[-\frac{\cos\theta_B}{f'_Y} M(q,k)_{\nu\lambda}^{338} + \frac{\sin\theta_Y}{f'_B} M(q,k)_{\nu\lambda}^{330}\right] \qquad (B.6)$$

$$T_{\nu\lambda}^{\omega\to\rho\pi} = -\frac{(k^2 - m_\omega^2)(k_1^2 - m_\rho^2)}{f_\pi\, a_\omega\, g_\rho} \left[\frac{\sin\theta_B}{f'_Y} M(q,k)_{\nu\lambda}^{338} + \right.$$

$$\left. + \frac{\cos\theta_Y}{f'_B} M(q,k)_{\nu\lambda}^{330}\right] \qquad (B.7)$$

$$T_{\nu\lambda}^{\phi\to\rho\pi} = -\frac{(k^2 - m_\phi^2)(k_1^2 - m_\rho^2)}{f_\pi\, a_\phi\, g_\rho} \left[-\frac{\cos\theta_B}{f'_Y} M(q,k)_{\nu\lambda}^{338}\right.$$

$$\left. + \frac{\sin\theta_Y}{f'_B} M(q,k)_{\nu\lambda}^{330}\right] \qquad (B.8)$$

The explicit momentum dependence of the function $M(q,k)_{\nu\lambda}^{ab8}$ is given by equations (2.13) and (2.24). $M(q,k)_{\nu\lambda}^{abo}$ can be obtained by making the replacements (2.17) using this information in equations (B.4)-(B.8) a little algebraic effort leads to the results quoted in equations (2.29)-(2.33).

APPENDIX C

In this appendix we give the details of the derivation of Eq. (2.36).

Consider the production of the state $X(p_X)$ in the reaction $e\gamma \to eX$ via one photon exchange process, see Fig. 1(c). The S-matrix element for this is

$$S_{fi} = \frac{e\, m_e}{(2\pi)^{9/2}\sqrt{E_1 E_2}\,\sqrt{2K_o}}\ \bar{u}\,(p_2)\ \gamma^{\mu}\ u(p_1)\ \frac{1}{k_1^{\ 2}}\, M_{\mu\alpha}$$

$$\epsilon^{\alpha}(k,\ \lambda)\ i(2\pi)^4\ \delta(p_1 + k - p_2 - p_X) \qquad (C.1)$$

where $\epsilon^{\alpha}(k,\ \lambda)$ is the photon polarization vector, and $M_{\mu\alpha}$ describes the $\gamma - \gamma^* - X$ vertex. γ^* is the virtual photon. $M_{\mu\alpha}$, which depends on k and k_1, satisfies $k_1^{\mu}\ M_{\mu\alpha} = 0$. Using this property and (C.1) the cross-section integrated over the final electron phase space can be written as,

$$d\sigma_{e\gamma \to eX} = \frac{\alpha}{2\pi^2}\ \frac{1}{2k_o E_1}\ \{ \int \frac{d^3 p_2}{E_2 k_1^{\ 4}}\ [2p_1^{\mu}\ p_1^{\nu} + \frac{1}{2}\ k_1^{\ 2}\ g^{\mu\nu}](-\ \frac{1}{8}\ M_{\mu}^{\beta}\ M_{\nu\beta}^{\dagger}\ )d\Gamma' \} \qquad (C.2)$$

where

$$d\Gamma' = (2\pi)^4\ \delta(k + k_1 - p_X)\ d\Gamma \ ; \ k_1 = p_1 - p_2$$

and $d\Gamma$ is the invariant phase space for the state X.

Now let us consider the production of X through two photon annihilation process ee → ee X, see Fig. 1(b). The amplitude is,

$$S_{fi} = \frac{e^2 m_e^2}{(2\pi)^6 \sqrt{E_1 E_1} \sqrt{E_2 E_2'}} \bar{u}(p_2) \gamma^\mu u(p_1) \frac{M'_{\mu\sigma}}{k_1^2} \frac{g^{\sigma\alpha}}{k^2}$$

$$\bar{u}(p_2') \gamma_\alpha u(p_1') i(2\pi)^4 \delta(k + k_1 - p_X) \quad \text{(C.3)}$$

where,

$$k^2 \simeq -2E_1' E_2' (1-\cos\theta') - \frac{m_e^2 (E_1' - E_2')^2}{E_1' E_2'} \quad \text{(C.4)}$$

xxxs $\cos\theta' = \hat{p}_1' \cdot \hat{p}_2'$

and $M'_{\mu\sigma}$ describes the $\gamma^* - \gamma^* - X$ vertex. In the centre of mass frame ($E_1 = E_1' = E$) the cross-section can be written as,

$$d\sigma_{ee \to ee X} = \left(\frac{\alpha}{2\pi^2}\right)^2 \frac{1}{E^2} \int \frac{d^3p_2}{E_2} \frac{d^3p_2'}{E_2'} \frac{1}{k_1{}^4} [2p_1^\mu p_1^\nu + \frac{1}{2} k_1^2 g^{\mu\nu}]$$

$$\frac{1}{8} M'_{\mu\sigma} M'^{\dagger}_{\nu\sigma'} \frac{g^{\sigma\alpha}}{k^2} \frac{g^{\sigma'\beta}}{k^2} [2p'_{1\alpha} p'_{1\beta} + \frac{1}{2} k^2 g_{\alpha\beta}] \, d\Gamma'$$

(C.5)

Now leading contribution to $d\sigma_{ee \to ee X}$ can be obtained by performing the photon polarization sum in the Coulomb gauge. Thus we make the following substitution in (C.5):

$$\frac{g^{\sigma\alpha}}{k^2} \rightarrow \frac{-g^{o\sigma}\ g^{o\alpha}}{\vec{k}^2} - \sum_{i=1,2} \frac{g^{i\sigma}\ g^{i\alpha}}{k^2}$$

The polarization directions i are orthogonal to $\hat{k}$. Ignoring the longitudinal contribution Eq. (C.5) becomes,

$$d\sigma_{ee \rightarrow ee\,X} = \frac{\alpha^2}{(2\pi^2)^2} \frac{1}{E^2} \int \frac{d^3p_2'}{E_2'} \frac{d^3p_2}{E_2} \frac{1}{k_1^4} [2p_1^\mu\ p_1^\nu + \frac{1}{2} k_1^2\ g^{\mu\nu}]$$

$$\frac{1}{8} \sum_{i,j=1,2} M'_{\mu i}\ M'^{\dagger}_{\nu j} \quad \frac{1}{k^4} [2p'_{1i}\ p'_{1j} - \frac{1}{2} k^2\ \delta_{ij}]\, d\Gamma' \tag{C.6}$$

Averaging over the azimuthal angle ϕ' of $\bar{p}_2'$ Eq.(C.6) reduces to

$$d\sigma_{ee \rightarrow ee\,X} = \frac{\alpha^2}{(2\pi^2)^2} \frac{1}{E^2} \int \frac{d^3p_2'}{E_2'\ k^4} [- \frac{k^2}{2} + \frac{E^2 E_2'^2 \sin^2\theta'}{\vec{k}^2}]$$

$$\{ \int \frac{d^3p_2}{E_2} \frac{1}{k_1^4} [2p_1^\mu\ p_1^\nu + \frac{1}{2} k_1^2\ g^{\mu\nu}]$$

$$\frac{1}{8} \sum_{i=1,2} M'_{\mu i}\ M'^{\dagger}_{\nu i} \quad d\Gamma' \} \tag{C.7}$$

Now $\frac{1}{8} \Sigma\, M'_{\mu i}\ M'^{\dagger}_{\nu i}$ depends on θ' through its dependence on k. So if we approximate this quantity by its value at $k^2 = 0$, we can substitute the curly bracket in (C.7) by that in (C.2) and obtain,

$$d\sigma_{ee \to ee\,X} = \frac{\alpha}{2\pi^2 E} \int \frac{d^3 p_2'}{E_2'} \frac{1}{k^4} \left[\frac{k^2}{2} - \frac{E^2 E_2'^2}{\bar{k}^2} \sin^2 \theta'\right]$$

$$2k_o \, d\sigma_{e\gamma \to eX}$$

Using (C.4) and integrating over $\cos \theta'$ this gives

$$d\sigma_{ee \to ee\,X} = \int_o^E \frac{dk_o}{k_o} N(k_o) \, d\sigma_{e\gamma \to eX}$$

where $k_o = E - E_2'$. From this Eq. (2.36) is easily obtainable.

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47. The asymptotic expression for K^2 and K_0 are for general point in α, β, β' space. On specific surfaces lines or points (e.g. when $\beta = -1/2$) they may assume less dominant behavior.

48. If there is a singularity of g_1 either at $\beta = -1/2$ or at $\alpha = \kappa'(\beta + 1/2)$ then as a consequence the asymptotic behavior of K^2 and K_0 are of one order less and the resultant expression for the cross section will be independent of dimension d. Further there could be uncompensated singularity of g_1 in either β or β' or when β and β' overlap, all of which result in the divergence of the integral in Eq.(5.13). We do not enter into a speculation whether such singularities might ruin the connection between dimension of pion source current and the asymptotic behavior of processes considered here.

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